

# On the Exploitation of CDF based Wireless Scheduling

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**Abstract**—Channel-aware scheduling strategies - such as the CDF Scheduler (CS) algorithm for the CDMA/HDR systems - provide an effective mechanism for utilizing the channel data rate for improving throughput performance in wireless data networks by exploiting channel fluctuations. A highly desired property of such a scheduling strategy is that its algorithm will be stable, in the sense that no user has incentive "cheating" the algorithm in order to increase his/her channel share (on the account of others). We present a scheme by which coordination allows a group of users to *gain permanent increase* in both their time slot share and in their throughput, on the expense of others, by misreporting their rates. We show that for large populations consisting of regular and coordinated users in equal numbers, the ratio of allocated time slots between a coordinated user and a regular one converges to  $e - 1 \approx 1.7$ . Our scheme targets the very fundamental principle of CS (as opposed to just attacking implementation aspects), which bases its scheduling decisions on the Cumulative Distribution Function (CDF) of the channel rates reported by users. Our scheme works both for the continuous channel spectrum and the discrete channel spectrum versions of the problem.

## I. INTRODUCTION

High-speed wireless networks are becoming increasingly common and along with that the strategy of *scheduling* the high-speed data - which is vital to the performance of modern wireless systems - has become the subject of active research. The modern wireless networks standards such as HSPDA [1] and EV-DO [2] [3] allows new generation of channel aware schedulers - such as the Proportional Fairness [4] [5] and the CDF scheduler [6] - which improve throughput performance by exploiting channel fluctuations while maintaining fairness among the users.

The CDF Scheduler (CS) makes scheduling decisions based on the Cumulative Distribution Function (CDF) functions of the users in such way that every time slot the user whose rate is the least probable to become higher is scheduled for transmission. An important property of this scheduler is that it statistically allocates all users an equal number of slots while smartly utilizing the knowledge of channel capacity to dynamically select at every moment the more attractive (higher capacity) users. A distinctive feature of this algorithm is that it allows to predict the exact throughput for each user based

on his/her<sup>1</sup> CDF alone, regardless of changes in the channel rate distribution of other users. These features and its simple notion of fairness (equal time share) make CS an attractive alternative to the Proportional Fairness Scheduler (PFS) [4]. Recent studies [7] [8] revealed the vulnerability of PFS to delays/jitter and loss of throughput caused by malicious users by providing false channel capacity reports. In this paper the vulnerability of the CDF scheduler to threats of non-conforming opportunistic users as well as malicious users is investigated for the first time.

One of the main roles of a resource allocation mechanism is to ensure fairness of the allocation under the assumption that every user aims at increasing his own allocation. Further, it is highly important that the scheduler will be resilient to users who may try to increase the resources allocated to them by not fully conforming with the protocol rules.

In this work we focus on the fundamentals of the algorithm, thus aiming at examining whether the algorithm can be attacked in a fundamental way, as opposed to attacking some of its implementation aspects.

We show that a *group of coordinated users* can *increase both the number of slots and the bandwidth* allocated to *each of them*. The capacity announcement strategy used by the coordinated users is very simple and requires only the knowledge of each other's capacity. We conduct the analysis of this strategy and derive its performance gains for both the continuous rate distribution model (Section III) and the discrete rate distribution model (Section IV). Our results show that the gain that such non-conforming users can achieve may be as high as 28% in a typical system configuration (30 users). Further, the ratio between the slot allocation of a coordinated user and a regular user can reach  $e - 1 \approx 1.7$ . We further consider coordinated *malicious* users. These aim at reducing the performance of the regular users, not caring about their own performance. We show that the channel share loss that the regular innocent users suffer can be as high as 48% in a typical system configuration.

The rest of the paper is organized as follows: After model and preliminaries given in Section II, Section III analyzes non-conformist users under the continuous rate distribution,

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<sup>1</sup>From now on we use "he" and "his" to mean "she/he" and "her/his" for the sake of reading flow.

and Section IV does it under the discrete rate distribution. In Section V we analyze the loss for regular users by coordinated and malicious users in the practical discrete model.

## II. ASSUMPTIONS, MODEL AND PRELIMINARIES

In the following we assume that time is slotted to slots  $t = 1, 2, \dots$ . Let  $R_k \geq 0$  be a random variable denoting the channel rate of user  $k$  (which is derived from his channel condition), and denote his channel rate at time slot  $t$  with  $R_k(t)$ . The CDF of  $R_k$  is  $F_{R_k}(r) = Pr[R_k \leq r]$ .  $R_k(t)$  is a stationary random process assumed to be independent of  $R_k(t')$  for any  $t \neq t'$  and of  $R_j(t')$  for any  $j \neq k$  and any  $t'$ .

At each slot  $t$ , each user  $k$  announces to the scheduler his actual value  $R_k(t)$ . The scheduler may compute the distribution  $F_{R_k}(r)$  from the past reports of user  $k$ . We demonstrate the vulnerability of CS without targeting a weak point in the CDF learning mechanism. Therefore throughout the paper we assume that the CDF learning mechanism of the scheduler has converged and thus it has full knowledge of the precise channel rates' CDF functions (as reported by the users). At time  $t$  the scheduler can use both the studied  $F_{R_k}(r)$  and the current user rates  $R_k(t)$  to decide to which user to transmit at slot  $t$ . The rate at which the server will transmit to the selected user, say  $k$ , is  $R_k(t)$ .

## III. THE BASIC PROBLEM: DEALING WITH CONTINUOUS RATE DISTRIBUTIONS

In this section we assume that all the channel rate distributions are continuous, that is the distribution functions do not contain mass values (i.e.  $F_{R_k}(r)$  is differentiable and  $Pr[R_k = x]$  equals zero for every  $x$ ). Later, in Section IV, we will deal with discrete (and mixed) probability functions.

### A. Scheduling Algorithm

The basic CDF Scheduler (CS), aiming at dealing with continuous distributions, was introduced in [6] and operates as follows. Recall that  $R_k(t)$  is the actual channel capacity of User  $k$  at time slot (TS)  $t$  and let  $k^*(t)$  be the user selected for data transmission. The scheduler selects  $k^*(t)$  to be the user for which  $P[R_k > R_k(t)]$  is the smallest among all users. That is, the user whose rate is the least probable to become higher, namely:

$$k^*(t) = \arg \max_k \{F_{R_k}(R_k(t))\},$$

where  $F_{R_k}(r) = P[R_k \leq r]$ . The original scheduler definition [6] includes the option to assign each user a special weight  $w_k$  according to  $k^*(t) = \arg \max_k \{F_{R_k}(R_k(t))^{\frac{1}{w_k}}\}$  but for the sake of simplicity we omit the weight factor and assume the Base Station (BS) serves the users equally. For notational simplicity we define  $V_k(t) = F_{R_k}(R_k(t))$ . Since  $k^*(t) = \arg \max_k \{V_k(t)\}$  we will refer to  $V_k(t)$  as the priority value assigned to User  $k$  at TS  $t$ .

### B. Coordinated Users Strategy

The idea of users reporting fake channel rate to exploit the properties of channel aware scheduler was already introduced in [8]. The users can fake channel rate by modifying their laptop's 3G PC cards, either using through the accompanying software development kit or through the device firmware. The providers cannot detect it, even if they attempt tamper-proof technique [8].

We now deal with a group of non-conforming opportunistic users who coordinate their channel rate reports in order to increase their slot and throughput shares. Let  $C$  be a group of  $|C| = L$  coordinated users and  $N$  be the number of additional regular users in the network. Each one of the coordinated users knows if his rate is the least probable to be higher (and therefore will get the highest scheduling priority) in  $C$  before reporting to the BS. Let  $c^*(t) = \arg \max_c \{F_{R_c}(R_c(t))\}$  where  $c \in C$  so each user knows if he is  $c^*$  or not.

The coordinated users strategy can be implemented using a simple coordinating medium/network that allows the users to share this little information. For example, a big factory using a designated private (low rate and cost) wireless network to coordinate the access points used by its employees in order to gain more throughput to its users.

The strategy is simple: at time slot  $t$  user  $c^*$  will be the only one acting normal (reporting his real channel rate  $R_{c^*}(t)$ ) while all others report zero.

### C. Analysis of the Coordinated Users Share

Let  $R'_c$  be the R.V. of the *reported* channel rate by User  $c$  when he follows the coordination strategy. Recall that  $R_c$  is the *real* channel rate of user  $c$ , so if the user behaved normally, we would have  $R'_c = R_c$ . The following Lemma computes the increase of the user's CDF value when following the strategy:

**Lemma 1.**

$$F_{R'_c}(r) = G(F_{R_c}(r)) \quad (1)$$

where  $G(x) = \frac{L-1}{L} + \frac{1}{L}x^L$  and  $c \in C$ .

The full proof is given in [9]. Equation (1) stems from: a) A fraction  $(L-1)/L$  of the time User  $c$  reports 0. b) The value reported (in the other  $1/L$  fraction of the time) is determined by taking the maximum over the  $F()$  values of the  $L$  users in  $C$ .

Recall that  $V_c(r) = F_{R_c}(r)$  is the priority value assigned to user  $c$  when reporting channel rate  $r$  to the BS when he behaves normally (always reporting his real channel rate). Now let  $V'_c(r) = F_{R'_c}(r)$  be the priority value he gets for  $r$  when following the coordination strategy, then according to Lemma 1 we get that:

$$V'_c(r) = G(V_c(r)). \quad (2)$$

**Theorem 1.** *A user that follows the coordination strategy will win every time slot that he would win when behaving normally; therefore the coordination strategy can only increase his throughput and time slot share.*

*Proof:* Assume user  $c_0$  obtains the highest priority value when all users behave normally, therefore  $c_0 = k^*(t) = \arg \max_k \{V_k(R_k(t))\}$ .

Now, since also  $c_0 = c^*(t)$ , then when C follows the coordination strategy,  $c_0$  will be the user who reports his real rate at this slot. Simple function analysis shows that  $G(x) > x$  for  $x \in (0, 1)$ , therefore according to Lemma 1 the priority value he gets  $V'_{c_0}(r) = G(V_{c_0}(r))$  is greater than  $V_{c_0}(r)$  which is greater than the priority values of all regular users. Therefore he still obtains this time slot when following the strategy. ■

**Theorem 2.** *In a network consisting of  $L + N$  users of which  $L$  are coordinated, the time share fraction dedicated to the  $L$  coordinated users (jointly) depends only on  $L$  and  $N$  (regardless of the channel rate distributions of any of users) and is given by:*

$$\frac{L}{N+1} \left( 1 - \left( \frac{L-1}{L} \right)^{N+1} \right).$$

The proof of Theorem 2 (which can be found in [9]) uses Lemma 1 to calculate the probability that one of the coordinated users will obtain a time slot.

This result now allows us to evaluate the inequality in time slot allocation between a coordinated user and a regular user. This inequality can be evaluated by the ratio between the slot shares of these users which, as shown next, can be very high:

**Corollary 3.** *Let  $C^{share}$  and  $R^{share}$  be the time share of the coordinated users and regular users respectively. When  $N = L - 1$ , then  $\lim_{L \rightarrow \infty} C^{share} = 1 - e^{-1}$  and  $\lim_{L \rightarrow \infty} R^{share} = e^{-1}$ . This means that  $\lim_{L \rightarrow \infty} \frac{C^{share}}{R^{share}} = e - 1 \approx 171\%$ . In contrast, this ratio under normal conditions equals  $L/N = L/(L-1)$  which converges to 100%.*

**Theorem 4.** *In a network consisting of  $L + N$  users of which  $L$  are coordinated, the average throughput (per time slot) of a coordinated user  $c$  is given by:*

$$\sum_{i=0}^N \binom{N}{i} \cdot \frac{(L-1)^{N-i}}{L^N} \int_{w=0}^1 w^{(i+1)L-1} \cdot F_{R_c}^{-1}(w) dw, \quad (3)$$

where  $F_{R_c}^{-1}(w)$  is the inverse function of  $F_{R_c}$ , the CDF of the real channel rate distribution.

The proof can be found in the full paper [9].

#### D. Evaluation and Discussion

In Figure 1 we evaluate the relative benefit in time slots a coordinated user gains from the coordination strategy; this is relatively to what he would get ( $\frac{1}{N+L}$ ) if he did not coordinate. The figure depicts this relative benefit as a function of the number of coordinated users  $L$  (given that the total number of users is fixed  $N + L = 30$ ). One may observe that the relative benefit is maximized at  $L = 11$ , implying that a coalition of 11 coordinated users has little incentive adding more users to the coalition. The relative benefit per user obtained is 28%.

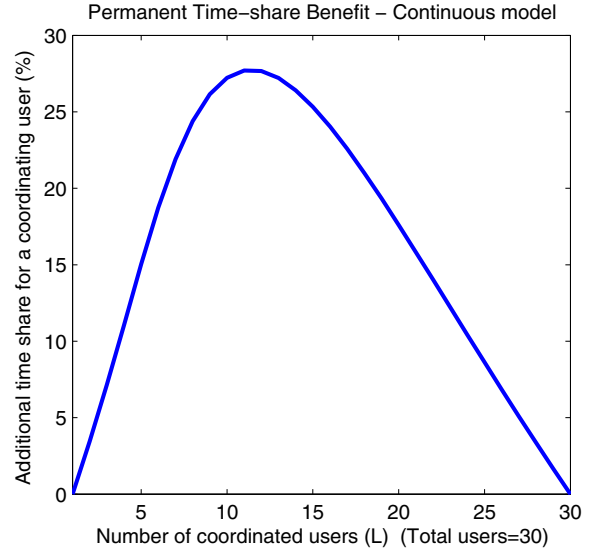


Fig. 1. Y-axis is the additional time share (in percents) that a coordinated user gains when he takes part in a coordinated group of size  $L$  (X-Axis).

When  $L = N + 1$  - which happens approximately at  $L = 15$  in Figure 1 - the time share of the  $L$  users when behaving normally is given by  $\frac{L}{2L+1}$  and converges to  $\frac{1}{2}$ . Corollary 3 pointed out that when  $L = N + 1$  the time share obtained by the coordinated users converges to  $1 - e^{-1}$  instead of  $\frac{1}{2}$ , meaning a coordinated user benefits from an additional time share of  $(1 - e^{-1})/(\frac{1}{2}) - 1 = 26.4\%$  which is close to the result at  $L = 15$  which equals 25.3%. Recall that according to Theorem 2 these results are valid for every system with 30 users regardless of the channel rate distributions of the users.

## IV. CS WITH DISCRETE CHANNEL RATES RANGE

### A. Scheduling Algorithm

The original version of the CS algorithm ([6]) assumed continuous channel rate values even though practical systems use discrete values. We now summarize the extension of the CS algorithm to the case of discrete channel rate values which appears in [10]. Again, to keep the calculations simple, we assume all users have the same weight and exclude the weight factor.

In the discrete model  $R_k(t) \in \{r_1, r_2, \dots, r_M\}$  where  $r_1 < r_2 < \dots < r_M$ . At TS # $t$  user  $k$  feeds back  $m_k(t) \in \{1, \dots, M\}$  the index of his channel rate value. Denote  $q_{k,m} \equiv F_{R_k}(r_m) = \sum_{i=1}^m P[R_k = r_i]$  where  $q_{k,0}$  is set to 0 for notational convenience. Instead of simply taking  $q_{k,m_k(t)}$  to be the priority value of user  $k$ , the CDF scheduler generates for each user a random priority given by a R.V.  $U_k(t)$  uniformly distributed in the interval  $[q_{k,m_k(t)-1}, q_{k,m_k(t)})$ , then the scheduler selects the user with the highest priority  $k^*(t) = \arg \max_k \{U_k(t)\}$ . The priority value of the discrete range algorithm ( $U_k$ ) preserves the fundamental character of the priority value of the continuous range algorithm ( $V_k$ ), namely that for every user  $k$  we get  $P[U_k(t) \leq x] = P[V_k(t) \leq x] = x$ .

## B. Coordinated Users Strategy

Assume a coordinated group of users  $C$  ( $|C| = L$ ) with the same channel rate probability (but independent from each other), namely  $\forall c_1, c_2 \in C \ q_{c_1, i} = q_{c_2, i}$ . Let  $m_c^*(t) = \arg \max_i \{r_i | \exists c. R_c(t) = r_i\}$  where  $c$  are users from the coordinated group. Every time slot, only the users with channel rate  $r_{m_c^*(t)}$  will report their real channel rate while all other report the lowest possible channel rate  $r_1$  (they have no chance getting the highest priority). The full analysis of the time and throughput share of the coordinated users can be found in the full paper [9].

## C. Evaluation and Discussion

While the share of a coordinated user in the continuous model depends only on the size of the coordinated group ( $L$ ) and the number of regular users ( $N$ ), the users' share in the discrete model depends also on the number of the possible channel rates ( $M$ ) and their probabilities among the coordinated users  $\{p_{c,i}\}_{i=1}^M$  (while still independent of the channel rate distributions of the regular users). The values of  $\{p_{c,i}\}_{i=1}^M$  in the system configuration considered in Figure 2 (a) were set according to the Rayleigh distribution on the 11 channel rates of the CDMA2000 1xEV-DO system in the same way which is described in [10] (where the CDF scheduler for discrete channel rates was presented).

As in Figure 1, Figure 2 (a) shows that the scheduler's notion of fairness is violated, we can see how the additional time share (Y-axis) for one coordinated user changes according to the number of users in the coordinated group ( $L$ ) in the same manner as for the continuous model (Figure 1) and for the same reasons that were already mentioned in section III-D.

Figure 2 (b) shows the throughput benefit. In order to compute the throughput, channel rate probabilities  $\{p_{c,i}\}_{i=1}^M$  were associated with actual rates in the CDMA2000 1xEV-DO as in [10]. For different sets of rates, we will get different results in Figure 2 (b), while Figure 2 (a) stays the same since it depends only on the set of probabilities  $\{p_{c,i}\}_{i=1}^M$  regardless of the actual rates associated with them. Note that the throughput benefit (as a function of  $L$ ) demonstrates similar behavior to the time share benefit.

## V. SYSTEM LOSS BY MALICIOUS STRATEGY (DISCRETE MODEL)

In the previous section we focused on non-conformist opportunistic coordinated users whose objective is to increase their own share of the network resources. This increase was, of course, accompanied by performance degradation to the regular innocent users which can be easily computed from our results in Sections III-D and IV-C.

Our interest in this section is in malicious users whose objective is only to damage the other regular users, disregarding their own performance. Of course – the malicious users can damage the innocent users at least as much as opportunistic coordinated users can do. So the major question addressed in this section is whether they can inflict greater damage, and by how much. Our focus in this analysis will be on the discrete

distribution model which, due to its practicality, is of higher interest (than the continuous model) to system designers.

## A. The Malicious Strategy

Assume a group of malicious users  $MAL = \{1, 2, \dots, S\}$  (for simplicity we assume their indices are  $1, 2, \dots, S$ ). The basic idea followed by the malicious users is to take turns, in a round-robin fashion, in trying to obtain time slots. This means that at TS  $t$ , malicious user number  $(t \bmod S)$  will attempt to obtain the time slot. Consider malicious user  $s$ : In all times slots that he tries to obtain he always reports the same channel rate - which we denote by  $r_{h_s}$  ( $h \geq 2$ ), while in all other slots he reports other rates which are all lower than  $r_{h_s}$ . Each user  $s$  chooses his  $r_{h_s}$  value independently of other users. This flexibility in choosing channel rates makes the malicious pattern very hard to detect.

## B. Analysis of the Malicious Strategy

**Theorem 5.** *In a network consisting of  $S + N$  users of which  $S$  use the malicious strategy, the time share fraction dedicated to the  $S$  malicious users (jointly) depends only on  $S$  and  $N$  and equals:*

$$\frac{S}{N+1} \left( 1 - \left( \frac{S-1}{S} \right)^{N+1} \right). \quad (4)$$

The proof of Theorem 5 can be found in the full paper [9].

**Remark 1.** *The damage (to the innocent users) inflicted by the malicious users under the discrete model is identical to the damage inflicted by the coordinated users strategy under the continuous model.*

The intuitive explanation for Remark 1 is that in both cases there is always exactly one user reporting his real channel rate (while the rest of the group are reporting zero)<sup>2</sup>.

## C. Evaluation and Discussion

Figure 3 demonstrates the (relative, percent-wise) time share loss experienced by each of the innocent users (compared to what he would get in a normal system) as a function of the number of malicious users ( $x$ -Axis). This is done for both the coordinated user strategy (dotted line) and the malicious strategy (solid line) in the discrete model. The channel distribution used is the Rayleigh distribution in CDMA2000 as in Figure 2. The loss caused by malicious users is significantly higher (by approximately a factor of 2) than that inflicted by the coordinated user strategy (for the discrete model). Note that according to Remark 1 we can use Corollary 3 to get an estimation for the time share loss of regular users. When  $S = N - 1$  the time share of the regular users converges to  $e^{-1}$  instead of 0.5, namely the loss of the regular users converges to 26% which is not far from the loss when  $S = N = 15$ , which equals 25% according to Figure 3.

Note that all the results in our work regarding malicious and coordinated users are independent of the channel rate

<sup>2</sup>Unlike coordinated users under the discrete model, where the number of coordinated users reporting their real rate could be greater than 1.

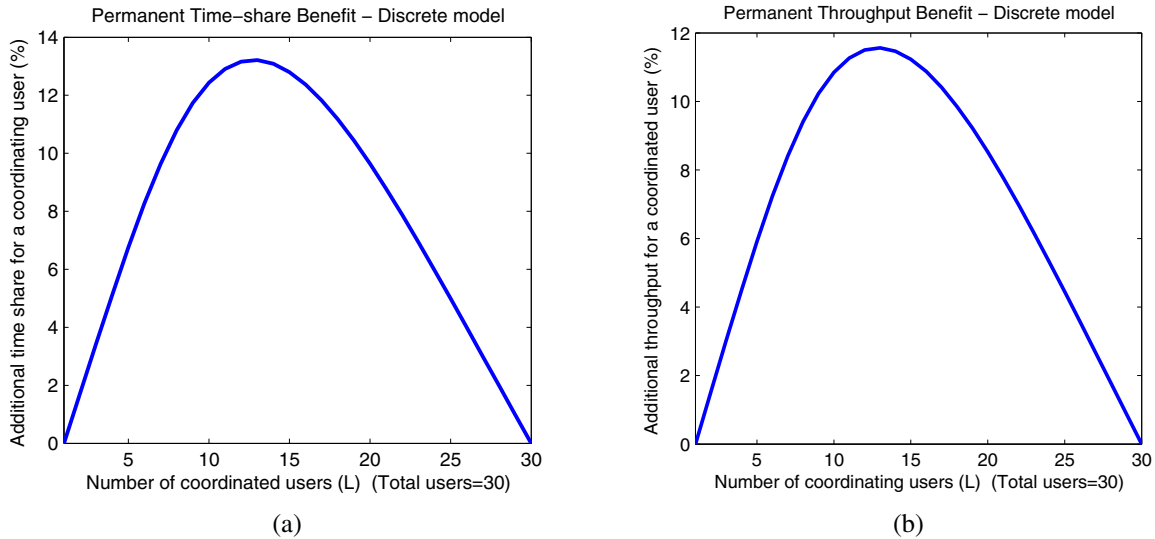


Fig. 2. The increase (in percents) of the time share (a) and throughput (b) that a coordinated user (experiencing Rayleigh fading channel in CDMA2000 1xEV-DO) gains when he participates in a coordinated group of size  $L$  (X-Axis). The results in both graphs show that – when there are 30 users in the system – participating in a coordinated group of 13 users is the most beneficial and increases the time share of the user by 13% and increases the throughput of each user by 11.6%.

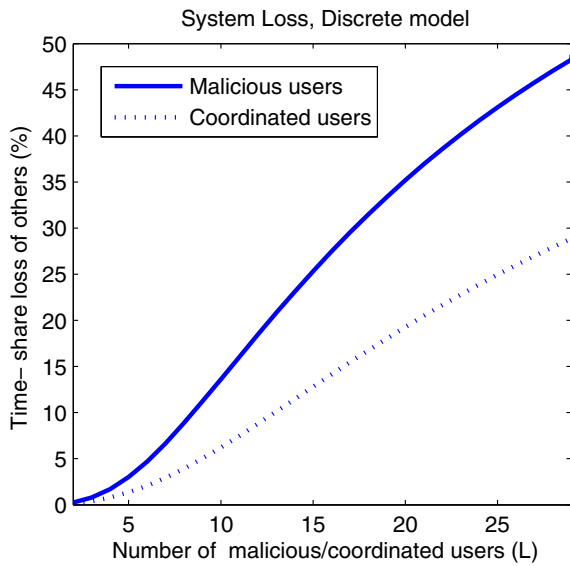


Fig. 3. Time Slot loss of innocent users: Malicious vs. Coordination strategy. The network's population is 30 users.

distributions of regular users. While the throughput loss of a regular user depends on his distribution, the case where some regular user experiences constant channel rate shows that the throughput loss can be as high as the time share loss.

## VI. CONCLUSION

In this paper, based on scheme which targets the very fundamental principle of the CDF scheduler, we showed that non-conforming opportunistic users have the motivation to misreport their channel rates and destabilize the scheduler's notion of fairness. In addition, we studied the loss for regular users, as inflicted by malicious users aiming at degrading the

system performance. We showed that for large populations consisting of regular and coordinated users in equal numbers, the ratio of allocated time slots between a coordinated user and a regular one converges to  $e - 1 \approx 1.7$ . After researches have proved the vulnerability of the Proportional Fairness scheduler, our work now demonstrates the vulnerability of its alternative – the CDF scheduler. This should be taken into consideration by system designers when choosing a scheduler for modern HDR wireless networks.

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