Lecture 4:

Forward Contracts and Forward Rates

04-1

Concepts and Buzzwords

- Forward Contracts
- Forward Prices
- Forward Rates
- Information in Forward Rates

- Buzzwords
  - settlement date, delivery, underlying asset
  - spot rate, spot price, spot market
  - forward purchase, forward sale, forward loan, forward lending, forward borrowing, synthetic forward
  - expectation theory, term premium

04-2

Readings:

- Tuckman, Chapter 2 and pp. 171-177, 180-182

04-3

Forward Contracts

- A ***forward contract*** is an agreement to buy an asset at a future *settlement date* at a *forward price* specified today.
  - No money changes hands today.
  - The pre-specified forward price is exchanged for the asset at the settlement date.

04-4

Forward vs. Spot

- To distinguish ordinary transactions from forward transactions, we use the word "spot" (or sometimes, "cash").
- A spot transaction is for settlement immediately.
  - Money and securities change hands today.
  - The price for such a transaction is called the "spot" price (or sometimes, "cash" price).

04-5

Motivation

- Suppose today, time 0, you know you will need to do a transaction at a future date, t.
- One thing you can do is wait until time t and then do the transaction at prevailing market prices
  - i.e., do a *spot* transaction in the *future.*
- Alternatively, you can try to lock in the terms of the transaction today
  - i.e., arrange a *forward* transaction *today*.

04-6

What is the fair forward price?

- In some cases, the forward contract can be **synthesized** with transaction in the current spot market.
- In that case, no arbitrage will require that the contractual forward price must be the same as the forward price that could be synthesized.

04-7

I. Synthetic Forward Price

- For example, if the underlying asset doesn't depreciate or make any payments, the synthetic forward price of the asset is

  Spot Price + Interest to settlement date
- How to synthesize?
  - **Buy the asset** now for the spot price.
  - **Borrow** the amount of the spot price, with repayment on the settlement date

  → You pay nothing now, and you pay the spot price plus interest at the settlement

04-8

Synthetic Forward Price for a Zero Bond

Suppose the underlying asset is $1 par a zero maturing at at time *T*.

In the forward contract, you agree to buy this zero at time *t*.

The forward price you could synthesize is

Spot Price + Interest to time *t*:

$$F_t^T = d_T \times (1 + r_t / 2)^{2t}$$

If the contractual forward price differs, there is an arbitrage opportunity.

04-9

Example:

Suppose the underlying asset is $1 par of a zero maturing at time *T=1*.

In the forward contract, you agree to buy this zero at time *t=0.5*.

The synthetic forward price is Spot Price + Interest

$$F_t^T = d_T \times (1 + r_t / 2)^{2t}$$

$$F_{0.5}^1 = d_1 \times (1 + r_{0.5} / 2)^{2\times 0.5}$$

= 0.9476 x (1+0.0554/2)

= 0.9739

What if the contractual forward price were 0.98?

04-10

Forward Contract as Forward Loan

- Just as we can think of the spot purchase of a zero as lending money, **we can think of a forward purchase of a zero as a "forward loan**."
- The forward lender agrees today to lend a pre-specified amount, at a pre-specified "forward rate," starting at a future date *t*, and ending at a later date *T*.
- This is the same transaction as a forward purchase of a zero maturing at time *T* with settlement time *t*. Just the language is different. The contract is usually specified in terms of a "forward rate" instead of a "forward price."

04-11

II. Forward as a portfolio of zeros

- You agree today (*t=0*) to pay at *t* the sum *$F* to get *$1* worth of par at *T*
- This contract is a portfolio of cash flows:

```
$0                -$F               +$1
|-----------------|-----------------|
0                 t                 T
```

- *What is the PV of this contract?*

It is a portfolio: ……………………………. V=
Long $1 par of T-year zeros …………….. +1 x d_T
Short $F par of t-year zeros……………… -F x d_t

04-12

Zero Cost Forward Price

- At *t=0* the contract "costs" zero
- The forward price should take care of that
- What is such "F"?

```
$0                  -$F                 +$1
|-----------------|-----------------|
0                   t                   T
```

- $V=0= -F \times d_t + 1 \times d_T$

➔ $F= d_T / d_t$

Or

$$d_T \times (1+r_t/2)^{2t} = F_t^T$$

04-13

Example

- Suppose the underlying is $1 par of the zero maturing at T=1 for settlement at t=0.5
- What is the value fo the contract if the forward price is *F=0.9738* ?
- *V= -0.9738 x 0.9730 + 1 x 0.9476 = 0*
- Arbitrage:
  - What if the forward rate you get quoted is F=0.98 ?
  - *V=-0.98x0.9730+1x0.9476=-0.0059*
  - What would you do?

04-14

III. Synthetic Forward Loan

- A forward loan can be synthesized by combining spot lending and spot borrowing.
- How? Today, at time 0,
  - borrow money to be repaid time t.
  - lend the *same present value* to time T.
    - *Today, time 0, there is no net cash flow.*
    - *At time t, you pay money.*
    - *At time T, you get money.*
    - *The amounts are all pre-determined.*

    → *You have locked in a forward loan.*

04-15

Synthetic Forward Rate

- What interest rate do you lock in on your synthetic forward loan?
  - Say you borrow \$1 to be repaid time *t*.
  - Lend same \$1 to time *T*:

	0	t	T
Borrow:	+1	$-(1+r_t/2)^{2t}$	
Lend:	-1		$+(1+r_T/2)^{2T}$
Net:	0	$-(1+r_t/2)^{2t}$	$+(1+r_T/2)^{2T}$

04-16

Synthetic or Implied Forward *Rate*…

- The synthetic forward loan lasts for *T-t* years.
- The starting and ending values of the loan imply a synthetic forward rate , *f*:

Gross interest on loan

$$(1+f_t^T/2)^{2(T-t)} = \frac{(1+r_T/2)^{2T}}{(1+r_t/2)^{2t}}$$

04-17

Example

- At time 0, you can synthesize a forward loan from time t=0.5 to time T=1 by
  - borrowing $1 to be repaid at time 0.5 and
  - lending $1 to be repaid at time 1.
  - The 0.5-year interest rate is 5.54%
  - The 1-year rate is 5.45%

Transaction	Time 0 cash flow	Time 0.5 cash flow	Time 1 cash flow
Borrowing	+1	-(1+0.0554/2)	
Lending	-1		$+(1+0.0545/2)^2$
Net	0	-(1+0.0554/2)	$+(1+0.0545/2)^2$

Your implied lending rate is the forward rate from 0.5 to 1:

$$(1+f_{0.5}^1/2) = \frac{(1+0.0545/2)^2}{(1+0.0554/2)^1} \Rightarrow f_{0.5}^1 = 5.36\%$$

04-18

Easier Way to See the Forward Rate Relation

We can lend from time 0 to t at rate r_t by buying a t – year zero spot, and we can guarantee lending from time t to T at rate f_t^T (by buying a forward contract), so we can lock in lending from time 0 to T.

The future value of each dollar invested would be:

$$(1+r_t/2)^{2t} \times (1+ f_t^T / 2)^{2(T-t)}$$

But we can also lock in lending from time 0 to T at rate r_T (by buying a T- year zero spot). That way the future value of each dollar invested would be

$$(1+r_T/2)^{2T}$$

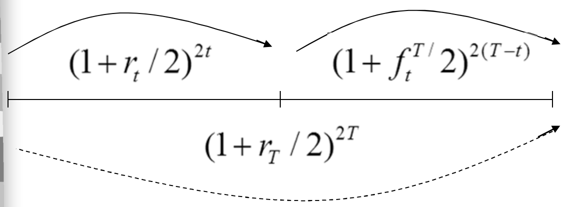
The two ways of creating a riskless loan from time 0 to T must be at the same lending rate:

$$(1+r_t/2)^{2t} \times (1+ f_t^T / 2)^{2(T-t)} = (1+r_T/2)^{2T}$$

04-19

Example

$$(1 + r_t / 2)^{2t} \times (1 + f_t^T / 2)^{2(T-t)} = (1 + r_T / 2)^{2T}$$



In the case of the forward rate from time t = 0.5 to time T=1:

$$(1+0.0554/2)^1 \times (1+0.0536/2)^1 = (1+0.0545/2)^2$$

04-20

Connection Between Forward Prices and Forward Rates

- What is the discount factor associated with the forward rate?

$$\frac{1}{1+f_t^T/2)^{2(T-t)}} = \frac{(1+r_t/2)^{2t}}{1+r_T/2)^{2T}}$$

$$= d_T \times (1+r_t/2)^{2t}$$

$$= \frac{d_T}{d_t}$$

$$= F_t^T$$

04-21

Example

- The implied forward rate for a loan from time 0.5 to time 1 is 5.36%.
- This gives a discount factor of 0.9739, which we showed before is the synthetic forward price to pay at time 0.5 for the zero maturing at time 1.

$$F_t^T = \frac{d_T}{d_t} = \frac{(1+r_t/2)^{2t}}{(1+r_T/2)^{2T}} = \frac{1}{(1+f_t^T/2)^{2(T-t)}}$$

$$F_{0.5}^1 = 0.9739 = \frac{1}{(1+0.0536/2)^1} = \frac{1}{1+f_{0.5}^1/2)^1}$$

04-22

Summary: One Equation, Many Economic Interpretations:

Forward price = Spot price + Interest

(1) $F_t^T = d_T \times (1 + r_t/2)^{2t}$

Present value of forward contract cash flows at inception = 0:

(2) $-d_t \times F_t^T + d_T \times 1 = 0$

Lending short + Rolling into forward loan = Lending long:

(3) $(1 + r_t/2)^{2t} \times (1 + f_t^T/2)^{2(T-t)} = (1 + r_T/2)^{2T}$

If we use the relations between prices and rates,

$d_t = \frac{1}{(1 + r_t/2)^{2t}}$ and $F_t^T = \frac{1}{(1 + f_t^T/2)^{2t}}$, ← ***Error: should be 2(T-t)***

We can verify that these equations are all the same.

Other arrangements:

(4) $F_t^T = \frac{d_T}{d_t}$ and $(1 + f_t^T/2)^{2(T-t)} = \frac{(1 + r_T/2)^{2T}}{1 + r_t/2)^{2t}}$

04-23

Spot Rates as Averages of Forward Rates

- Rolling money through a series of short-term forward contracts is a way to lock in a long term rate and therefore synthesizes an investment in a long zero. Here are two ways to lock in a rate from time 0 to time t:

$$(1 + r_{0.5}/2) \times (1 + f_{0.5}^1/2) \times \cdots \times (1 + f_{t-0.5}^t/2) = (1 + r_t/2)^{2t}$$

- The growth factor *(1+r_t/2)* is the geometric average of the (1+f/2)'s and so the interest rate r_t is approximately the average of the forward rates.
  - Recall the example
    - The spot 6-month rate is 5.54% and the forward 6-month rate is 5.36%
    - Their average is equal to the one year rate of 5.45%

04-24

Forward Rates vs. Future Spot Rates

- The forward rate is the rate you can fix today for a loan that starts at some future date.
- By contrast, you could wait around until that future date and transact at whatever is the prevailing spot rate.
- Is the *forward rate* related to the random *future spot rate*?
- For example, **is the forward rate equal to people's expectation of the future spot rate**?

04-25

The Expectations Hypothesis

- The simple "Expectations Hypothesis" says that the forward rate is equal to the expected future spot rate.
- This may not necessarily hold, because people might require extra expected return to be willing to hold bonds that don't match their investment horizon.
- For example, the yield curve is generally upward sloping.
  - Does that mean people always expect rates to rise?
  - Or do investors simply require more expected return to be willing to hold longer bonds?

➔ Empirically, forward rates tend to be higher than the spot rate that ultimately prevails for that investment horizon.

04-26

Term Premiums and Information in Forward Rates

- The "term premium" is defined roughly by

  Forward rate = Expected future spot rate + Term Premium
- So, a more general version of expectations hypothesis says that term premiums are roughly constant (e.g., IR risk has a constant price)
- If that's true, then changes in forward rates reflect changes in expectations about future rates.
- For example, if we see forward rates fall, it may mean that people have revised their expectations about future spot rates downward.
- …on the other hand it could be because risk premiums have changed.

04-27

How to test the expectations hypothesis?

- What would you expect under the exp hyp if you run the following regression:

$$R_{t+k}^{1}=a+bF_{t}^{t-k-1,t-k}$$

- To avoid "statistical mess", Fama Bliss suggest running

$$R_{t+k}^{1}-R_{t}^{1}=a+b(F_{t}^{t-k-1,t-k}-R_{t}^{1})$$

04-28

Fama Bliss AER 1987

TABLE 3—REGRESSION FORECASTS OF THE CHANGE IN

Dependent	a	$s(a)$	b_1	$s(b_1)$	b_2	$s(b_2)$	R^2
$r(1:t+x-1)-r(1:t)=a+b_1[f(x,x-1;t)-r(1:t)]+u(t+x-1)$							
$r(1:t+1)-r(1:t)$	.21	.41	.09	.28			.00
$(1:t+2)-r(1:t)$	.40	.73	.69	.26			.08
$r(1:t+3)-r(1:t)$	.57	.75	1.30	.10			.24
$r(1:t+4)-r(1:t)$	1.12	.61	1.61	.34			.48

04-29

http://www.faculty.idc.ac.il/kobi/XREH.pdf

THE INFORMATION IN LONG-MATURITY FORWARD RATES: IMPLICATIONS FOR EXCHANGE RATES AND THE FORWARD PREMIUM ANOMALY

Jacob Boudoukh,[a] Matthew Richardson,[b] and Robert F. Whitelaw[b*]

Table 4: The Expectations Hypothesis of Interest Rates

Country	j	α	Std. err.	β	Std. err.	R^2
US	1	–0.37	0.33	0.00	0.25	0.00
	2	–0.72	0.81	0.13	0.45	0.30
	3	–1.39	1.12	0.70	0.38	7.04
	4	–2.06	1.12	1.16	0.24	19.15
UK	1	–0.24	0.29	0.47	0.22	8.84
	2	–0.78	0.56	1.00	0.27	25.78
	3	–1.03	0.71	1.18	0.31	33.92
	4	–1.47	0.71	1.40	0.33	46.18
Germany	1	–0.36	0.34	0.48	0.19	6.28
	2	–0.96	0.54	1.01	0.28	20.04
	3	–1.66	0.54	1.40	0.35	36.28
	4	–2.33	0.50	1.62	0.30	50.47