financial risk measurement & management

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Course outline

- **Introduction to VaR Feb16**
  » Statistical framework. Risk and diversification: some examples.
  Visual interpretation. Possible applications.
- **The Stochastic Behavior of Asset Returns Feb16,Feb23**
  » Time variations in volatility. VaR: approaches and comparison. The *Hybrid Approach* to VaR.
  » ***Quiz1***
- **Beyond Volatility Forecasting Feb23,Mar1**
  » The VaR of derivatives and interest rate VaR. Structured Monte Carlo. Extreme events and correlation breakdown. Stress testing and scenario analysis. Worst case scenario
  » Guest lecture: TBA
  » The Crisis
  » ***Quiz2***
  » ***Quiz3*** *will take place during the 1st class of Multinational Financial Management on March8*

Administration

Readings
Textbook: *(ABS)* ***Understanding Market, Credit, and Operational Risk: The Value at Risk Approach***; Linda Allen, Jacob Boudoukh and Anthony Saunders, Blackwell
Course material: copies of slides will be handed out and appear on my website.

Grading
•Final grade: 90% best 2 of 3 quiz, 10% class participation.
•There will be no makeup quiz exams. If you miss one exam for a "permissible" reason (sickness or reserve duty with appropriate paperwork) the grade will be ignored. If you miss an exam without a permissible reason the grade on this exam will be zero.
•To be clear, missing an exam for work-related reason does not qualify as a "permissible reason" even if you let me know in advance.
•Sample questions can be found at the back of this course packet.

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Risk categories

Inherent risks		
Primary risks	**Operational risks**	
Credit risk	Transaction processing risk	Compliance risk
Market risk	Legal risk	Liability risk
Liquidity and funding risk	Security risk	Tax risk

- Market risk
  - » interest rate, currency, equity, commodity, spread, volatility,…
  - » *example: Price of bond declines as interest rates rise*
- Credit risk
  - » default, downgrade
  - » *example: P(bond)=recovery upon default*
- Other
  - » Operational risk, liquidity risk, regulatory risk , political risk, model risk…

- We focus primarily on **Market Risk**, and to a lesser extent on credit risk

Risk Measurement

- Address the question:

" HOW MUCH CAN WE LOSE
ON OUR TRADING PORTFOLIO
BY TOMORROW'S CLOSE? "

- Risk MEASUREMENT **<=?=>** Risk MANAGEMENT

VaR: Example

Consider a spot equity position worth $1,000,000

- Suppose the daily standard deviation of the S&P500 is 100 basis points per day
- How do we make an informative statement about risk?

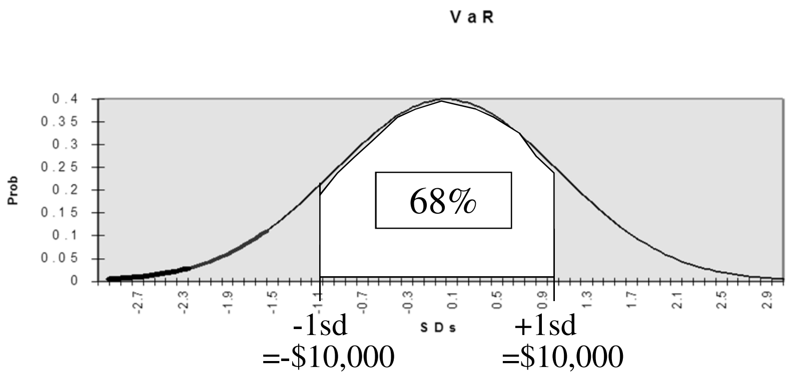
We can only make a probabilistic statement:

Assume $\Delta S_{t,t+1}$ is distributed normally (0 , 100_{bp}^2)

"Value at Risk" (VaR):

(First Look)

- From the normal dist'n tables:
  - » -1STD to +1STD 68.3%
  - » -2STD to +2STD 95.4%
  - » What is the "value" of one standard deviation?
  - » What are the ***amounts*** on the X-axis?



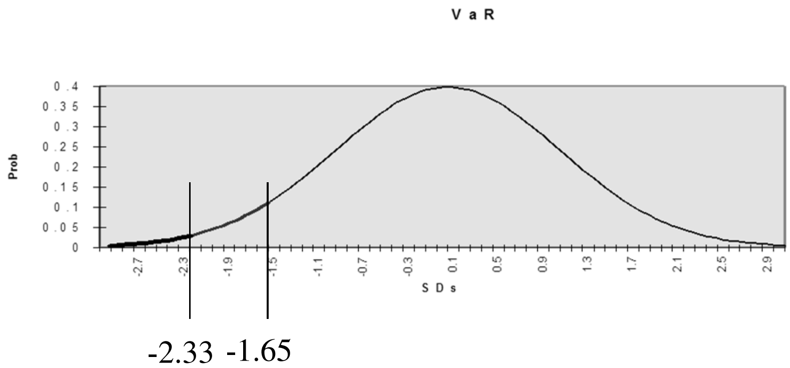
"Value at Risk" (VaR):

(First Look)

- From the normal dist'n tables:

  Prob(Z< -1.65)=5%, *Prob(Z*< -2.33)=1%

"With probability 95% we will not see a loss greater than ?_______? on our position"



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Quantifying the Exposure:
Calculating the standard deviation

- Three ways to define the change in the spot rate
  - » Absolute change $\Delta S_{t,t+1} = S_{t+1} - S_t$
  - » Simple rate of return $\Delta S_{t,t+1} = S_{t+1} / S_t$
  - » Cont' comp' change $\Delta S_{t,t+1} = ln(S_{t+1}/S_t)$
- Which definition of "$\Delta S_{t,t+1}$" is most appropriate?
- To answer this we must recognize that we are going to make a strong assumption:

the past is representative / predictive of the future

- The question is :

  which prism should we use to look back into the past?

Stationarity

- We are going to assume ***"stationarity"***
- Consider the following statements
  - » a 80pt change in the Dow is as likely at 6,000 as it is at 12,000
  - » a 2% change in the Dow is as likely at 6,000 as it is at 12,000

  ...which one is more likely to hold ?
- Recall: three ways to define the change in the spot rate
  - » Absolute change $\Delta S_{t,t+1} = S_{t+1} - S_t$
  - » Percentage change (rate of return) $\Delta S_{t,t+1} = S_{t+1} / S_t$
  - » C.C. change $\Delta S_{t,t+1} = ln(S_{t+1}/S_t)$

Time - consistency

- Consider the continuously compounded two-day return

$$\Delta S_{t,t+2} = ln(S_{t+2}/S_t) = ln\{(S_{t+2}/S_{t+1}) * (S_{t+1}/S_t)\} = ln(S_{t+2}/S_{t+1}) + ln(S_{t+1}/S_t) = \Delta S_{t,t+1} + \Delta S_{t+1,t+2}$$

- Suppose $\Delta S_{t+i,t+i+1}$ is distributed $N(0, \sigma^2)$

➔ The sum $\Delta S_{t,t+J}$ is also normal: $N(0, J*\sigma^2)$
(under certain assumption, to be discussed later)

- Easy to extrapolate VaR:

_J_ day VaR = SQRT(_J_) * (1 day VaR)

- With any other definition of returns normality is not preserved (i.e., the product of normals is non-normal)

Non-negativity

- Consider the cont' comp' *J*-day return

$$\Delta S_{t,t+J} = ln(S_{t+J}/S_t)$$

Since $\Delta S_{t,t+J}$ is distributed $N(0, J\sigma^2)$, the value of any possible S_{t+i} is guaranteed to be non-negative:

$$S_{t+i} = S_t \exp\{\Delta S_{t,t+J}\}$$

- This is the standard ***log-normal diffusion*** process (such as in Black/Scholes): ***log(*returns) are normal**
- With other definitions of returns positivity of asset prices is not guaranteed

Interest rates and spreads

- The exceptions to the rule are interest rates and spreads (e.g., zero rates, swap spreads, Brady strip spreads,…)
- For these assets the "change" is $\Delta i_{t,t+J} = i_{t+J} - i_t$, usually measured in basis points
- This is an added complication in terms of calculating risk
  - » for stocks, commodities, currencies etc, there is a ***1-for-1*** relation between the risk index and the portfolio value
  - » with interest rates there is a ***1-for-D*** relation, where D is the duration: a ***1***bp move in rates ==> ***D*** bp move in bond value

Quantifying the Exposure:
Calculating Standard deviation (cont'd)

- The STD of change can be calculated easily
  - » ***"Volatility"*** (vol)

$$STD(\Delta S_{t,t+1}) = SQRT\left[\ VAR(\Delta S_{t,t+1})\ \right]$$

  - » ...where $VAR(\Delta S_{t,t+1})$ is the ***"Mean squared deviation"***

$$VAR(\Delta S_{t,t+1}) = AVG\ [\ (\Delta S - avg(\Delta S))^2\ ]$$

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A two asset example

- Consider the following FX position

*(where vol($/Euro)=75bp ==> VaR=75*1.65=123,*

*vol($/GBP)=71bp ==> VaR=71*1.65=117*)

	Position *	95% move =	VaR
	(FX in $MM)	(in percent)	(in $MM)
Euro	100	1.23%	$1.23
GBP	-100	1.17%	-$1.17

Undiversified risk **$2.40**

(absolute sum of exposures, ignoring the effect of diversification)

Portfolio variance: a quick review

- X, Y are random variables *c, d* are parameters

$VAR(X+Y) = VAR(X)+VAR(Y)+2COV(X,Y)$

$VAR(c*X) = c^2\ VAR(X)$

==>

$VAR(cX+dY) = c^2\ VAR(X)+d^2\ VAR(Y)+2\ c\ d\ COV(X,Y)$

...applied to portfolio theory:

- A portfolio of two assets: $R_p= w\ R_a + (1-w)\ R_b$
  and the vol of a portfolio, $VAR(\ R_p\ )$, can now be calculated

Correlation and covariance

- **Correlation:** the tendency of two variables to co-move
  $COV(R_a, R_b) = \rho_{Ra,Rb} * \sigma_{Ra} * \sigma_{Rb}$

- The volatility of a portfolio in percent:
  $\%STD=sqrt[w_a^2\sigma_{Ra}^2 + w_b^2\ \sigma_{Rb}^2 + 2w_a w_b\ \rho_{Ra,Rb}\ \sigma_{Ra}\ \sigma_{Rb}\ ]$

- The volatility of a portfolio in \$ terms:
  $\$STD=sqrt[\$\sigma_{Ra}^2 + \$\sigma_{Rb}^2 + 2\ \rho_{Ra,Rb}\ \$\sigma_{Ra}\ \$\sigma_{Rb}\ ]$

From *vol* to VaR

How do we move from vol to VaR?

- Consider the $ volatility

$$\{ \$STD=sqrt[\$\sigma_{Ra}^2 + \$\sigma_{Rb}^2 + 2\ \rho_{Ra,Rb}\ \$\sigma_{Ra}\$\sigma_{Rb}\ ] \}*1.65$$

- and we get

$$\$VaR=sqrt\{\$VaR_{Ra}^2 + \$VaR_{Rb}^2 + 2\rho_{Ra,Rb}\ \$VaR_{Ra}\$VaR_{Rb}\}$$

- ...or we could calculate the %vol and

$VaR= \%STD * value * 1.65$

- The two approaches are EQUIVALENT

Portfolio VaR

- Suppose $\rho_{\$/Euro,\$/GBP} = 0.80$

- *For simplicity forget cont' comp' returns for the moment*

- $\$VaR=sqrt\{\$VaR_{Ra}^2 + \$VaR_{Rb}^2 + 2\boldsymbol{\rho_{Ra,Rb}}\ \$VaR_{Ra}\$VaR_{Rb}\}$

$=sqrt\{1.23^2 + (-1.17)^2 + 2*\mathbf{0.80}*(-1.17)(1.23)\ \}$

$=sqrt\{1.5129 + 1.3689\ \mathbf{- 2.3025}\ \}$

$= \$0.76Mil$

The portfolio effect

- Compare:

	Undiversified VaR	**$2.40MM**
	Diversified VaR	**$0.76MM**
==>	**Portfolio effect**	**$1.64MM**

- Risk reduction due to diversification depends on the **correlation** of assets in the portfolio
- As the number of assets increases, portfolio variance becomes more dependent on **covariances** and less dependent on variances
- The "marginal" risk of an asset when held in a small portion in a large portfolio, depends on its return **covariance** with other securities in the portfolio
- Exercise:
  - » Take an equally weighted portfolio with N uncorrelated asset.
  - » Assume all assets have equal volatility.
  - » What is the portfolio's volatility?
  - » Take N to infinity. What happens to volatility?

Diversification example: hedge funds

- Consider a fund manager examining 9 positions taken by hedge funds he invests in
- For simplicity assume that:
  - » the positions are valued at $100MM each, with a an annual VaR 16.5% (i.e., vol = 10% per annum)
  - » the strategies are *uncorrelated*

    (e.g., high yield FX, special situation, fixed income arb, Japanese warrants arb, spread trading,...)
- The undiversified VaR is *9*$16.5MM = $148.5MM*, on a *$900MM* investment

The funds' VaR

- The VaR of *N* uncorrelated assets:

VaR_{port} = $sqrt\{ 9 * VaR_{strat}^2 + zero\ covariance\}$

= $sqrt\{9\} VaR_{strat}$

= 3 * $16.5MM =$49.5MM

==> the risk reduction due to diversification is 66%

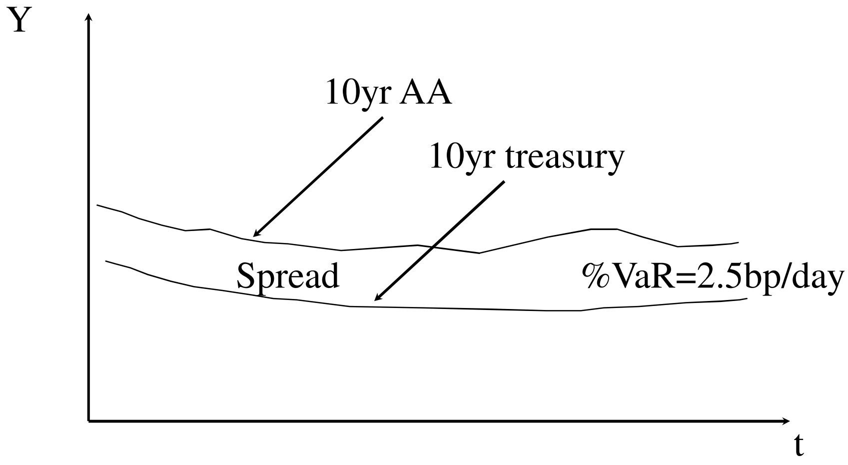
- Now suppose each strategy had an annualized Sharpe ratio of *E[R]/STD[R]=2 ==> E[R]=20%, or $180MM.*
- The portfolio's Sharpe ratio would be 180/30 = 6
- What would the Sharpe Ratio and VaR be if we invested the entire $900MM in only one strategy?

Bond portfolio VaR

- The VaR of a portfolio of $100 of face of a 1yr bond and a10yr bond can now be calculated as usual *(how?)*
- What is the VaR of a 10% s.a. coupon bond w/ 10yr to maturity?
- Note that the coupon bond VaR involves
  - » 20 volatilities
  - » 190 correlations

==> MATURITY BUCKETS
(cash flow mapping)

Spread VaR



The (approx) VaR of a 10yr AA bond

- *VaR(treasury)=11.5bp*
- *VaR(spread)=2.5bp*
- *CORR(Δspread, Δtreasury)=0*

==>*VaR(AA Bond)=sqrt{* $11.5^2 + 2.5^2$ *} = 11.77bp/day*

Note:
VaR(AA Bond) / VaR(treasury) = 11.77/11.5 = 1.023,
only 2.3% higher VaR!

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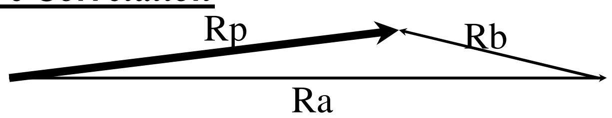
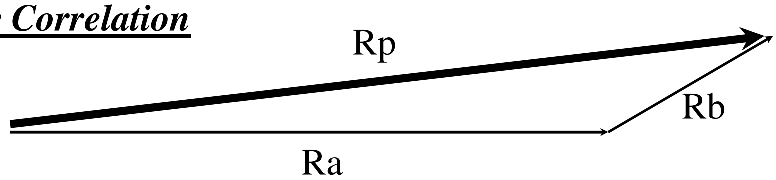
Visual interpretation

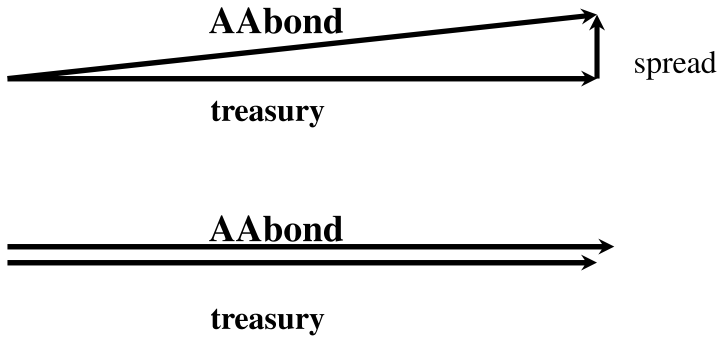
Negative Correlation



Positive Correlation

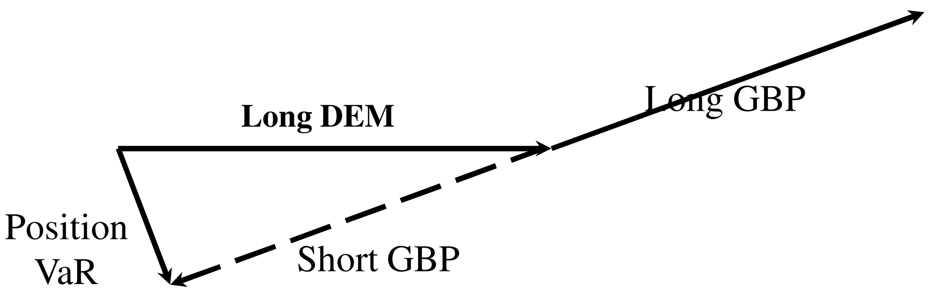


Visual interpretation - corp' bond example



Think, similarly, on the total risk of an FX equity investment

Visual interpretation - FX example



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Uses and applications

- Corporates
- Financial Institutions

Internal uses	External uses
- Trading limits	- Reporting
- Capital Allocation	- Capital requirements

- Self regulation

- **Self regulation and market disclosure**
  - » An alternative to the BIS's and the Fed's proposals, which may result in capital inefficiency and mixed incentives

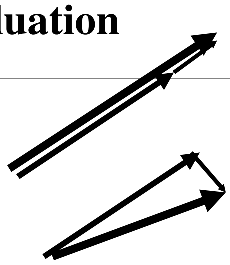
Trading Limits

- **VaR for management information and resource allocation**
  - » A unified measure of exposure at the trader, desk, group,... level
  - » An input for capital allocation and reserve decisions

ISSUES

- Full system may include VaR limits, notional limits, types of securities, types of exposures,…
- Implementation isn't simple

VaR and Performance Evaluation



- Idea:
  - » Desk1: *corr(P&L, IntRates)* close to one
  - » Desk2: *corr(P&L, IntRates)* close to zero
  - » *VaR1=VaR2*

  Performance/Compensation is a function of

  *P&L - C * VaR*

  where "C" is the price of risk parameter

  BUT marginal VaR contribution of Desk1 >> than Desk2

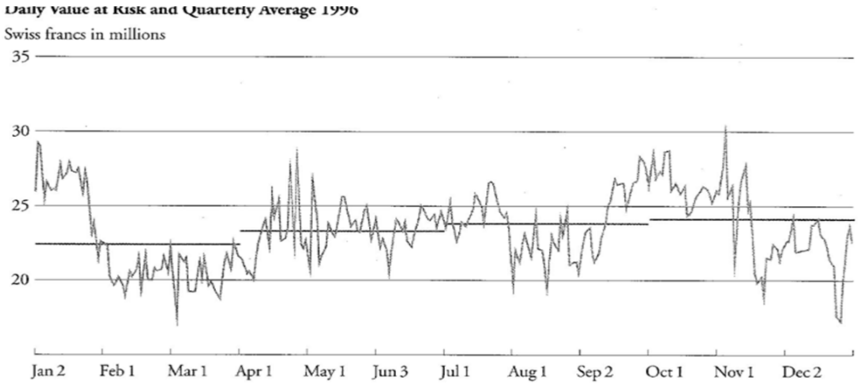
  ==> *P&L - C * **MarginalVaR***

  … and what if P&L=0 and MarginalVaR<0???

- In reality future P&L is a function of realized P&L and VaR or realized risk

VaR for Reporting:
example: UBS's VaR

- Time series of daily estimated VaR (defined as 2*STD) out of UBS's annual report



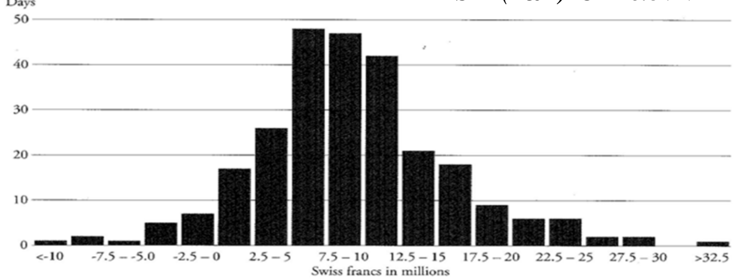
Source: 1996 annual report

UBS's realized P&L

- *Realized VaR=2*STD(P&L)=CHF13.6MM*

  << ExAnte VaR=CHF23.4MM
- *"...thereby underscoring the effectiveness of continuous risk management"*

AVG(P&L)=CHF9.2MM, STD(P&L)=CHF6.8MM



Source: 1996 annual report

Risk-type Allocation

Investment Bank: Value at Risk (10-day 99% confidence)

	Year ended 31.12.04			
CHF million	Min.	Max.	Average	31.12.04
Risk type				
Equities	121	188	153	126
Interest rates	244	441	340	361
Foreign exchange	5	73	30	29
Other[1]	9	87	37	32
Diversification effect	[2]	[2]	(202)	(215)
Total	274	457	358	332

1 Includes precious metals and energy exposures. 2 As the minimum and maximum occur on different days for different risk types, it is not meaningful

Business Group Risk Allocation

UBS: Value at Risk (10-day 99% confidence)

	As at 31.12.04	Year ended 31.12.04			
CHF million	Limits	Min.	Max.	Average	31.12.04
Business Groups					
Investment Bank	600	274	457	358	332
Wealth Management USA	50	12	27	17	16
Global Asset Management[1]	30	5	16	11	7
Wealth Management & Business Banking	5	1	1	1	1
Corporate Center[2,3]	150	35	69	47	38
Reserve	170				
Diversification effect		[4]	[4]	(69)	(62)
Total	750	274	453	365	332

1 Only covers UBS positions in alternative & quantitative investment funds. 2 The private label banks are included in Wealth Management & Busine
Center from 1 July 2003. 3 Includes interest rate exposures in the banking books of Treasury and, from 1 July 2003, the private label banks. 4 As th
for different Business Groups, it is not meaningful to calculate a portfolio diversification effect.

Regulatory Environment

- Objective: *"To provide an explicit capital cushion for the price risks to which banks are exposed, particularly those arising from their trading activities… important further step in strengthening the soundness and stability of the international banking system and of financial markets generally"* *(Jan96 amendment to the BIS capital accord)*
- The use of internal models will be conditional upon explicit approval of the bank's supervisory authority.
- Criteria
  - » **General criteria** re risk management systems
  - » **Qualitative criteria**
  - » **Quantitative criteria**
  - » Criteria for **external validation** of models
  - » **Stress testing**

Quant Standards

- No particular type of model required (e.g., VarCov, HistSim, SMC…)
- VaR on a daily basis
  - » 99th %ile
  - » 10 day horizon
  - » Lookback at least 1yr
- Discretion to recognize empirical corr within broad risk categories.
- VaR across these categories is to be aggregated (simple sum…)

Capital Requirements

CapRequ= MAX[VaR $_{t-1}$, *(Mult+AddOn)*AVG(VaR* $_{t, t-60}$*)]*

- Mult=3.
- AddOn related to past performance
  - » Green zone 4/250 exceptions 1%VaR OK
  - » Yellow zone up to 9/250 AddOn =0.3 To 1
  - » Red zone 10plus/250 Investigation starts
- Model must capture risk associated with options
- Must specify risk factors and a price-factor mapping process *a priori*

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The Stochastic Behavior of Asset Returns

- The problem of f-a-t t-a-i-l-s
  http://www.bloomberg.com/video/67293058/
- Time variations in volatility
- VaR: approaches and comparison
- The *Hybrid Approach* to VaR
- Long horizon VaR
- Benchmarking and backtesting VaR

How can we obtain the 5% tail move?

- So far the answer was: **VaR(5%)= 1.65 * σ**
- Asset returns are assumed to be

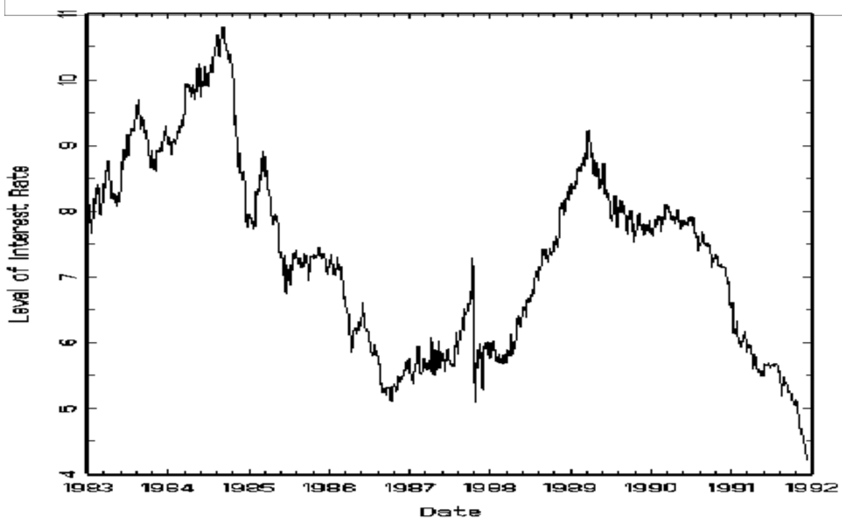
Stable ***...but vol varies through time***

and

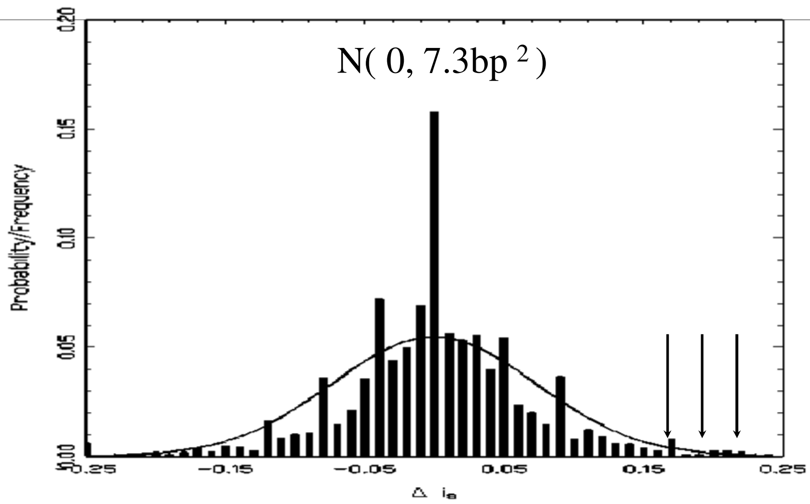
Normal ***...but they are not***

How do we make this determination?

3 months T-bill rate



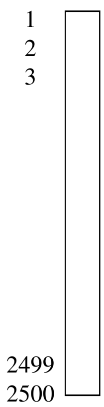
Interest rate changes: are they normal?



The Tails of the Distribution

- There ate, say 2500 Δι's
- Order them in ascending order
- The 1%-ile is, under normality
  *7.3*2.33=17bp*
- Where should you find this "17"?

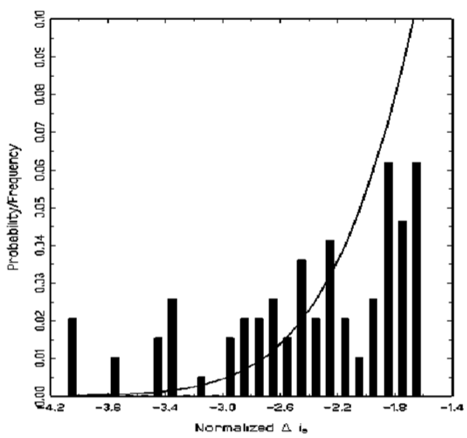
- What do you actually find there?



Fat tails

- If IR changes were normal: *Prob(IR change >17bp) = 1%*
- ... but in reality *Prob(IR change >21bp) = 1%*
  ===> "*F--A--T* Tails"
- This is especially true for return series such as oil, Bradys, some currencies...

- The Effect subsides gradually by aggregation
  - » through time
  - » cross sectionally



Why are tails so fat?

In trying to explain the fat tails, it could be the case that returns are simply fat tailed relative to the normal distribution or that returns are conditionally normal, but:

1. expectations vary through time

(well, maybe, but not enough to explain the tails)

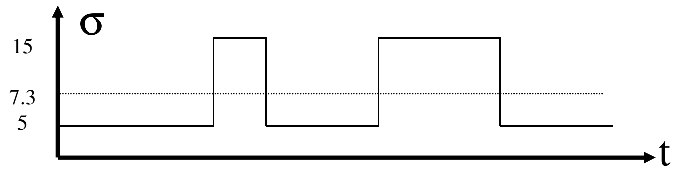
2. volatility varies through time

(and we know it does, but is it enough to explain the tails?)

PLAN: we follow 2 . . . but go back to 1

The Effect of Cyclical Vol.

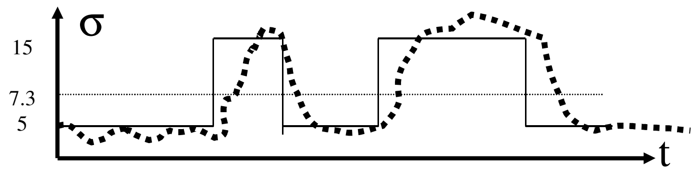
- We measure vol as 7.3bp/day
- Suppose now that in fact



- If in a given day *Δι=22bp*, then do we interpret is as *22/7.3=3sd,* or *22/15=1.5sd* ?
- This is the key goal of dynamic VaR engines

The Difficulty in Estimating Cyclical Vol.

- Need few days of data to realize change in volatility
- Key question: how adaptable do you want to be



- Tradeoff exists, and we shall elaborate on it next
- But in general, measuring volatility dynamically is the key goal of VaR engines
- … and it's very difficult

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  - » The VaR of derivatives and interest rate VaR. Structured Monte Carlo. Extreme events and correlation breakdown. Stress testing and scenario analysis. Worst case scenario

Modeling time-variations in VaR

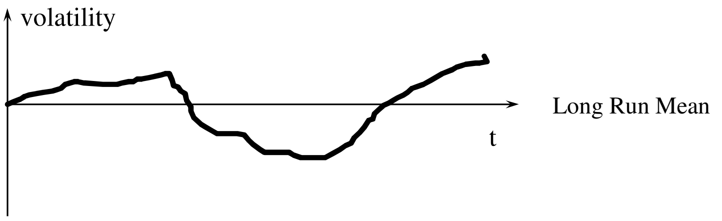
- **PARAMETRIC approaches**
  estimate the parameters of a given distribution
  - » STD - simple historical vol + Conditional normality
  - » Declining weights + Conditional normality (RiskMetrics)
  - » Mixture of normals, t-distribution, GARCH...
- **NONPARAMETRIC approaches**
  let the data talk
  - » Historical simulation
- **The *Hybrid Approach***

- **Finance-based forecasts** (e.g., implied vol)

Volatility is cyclical

Mandelbrot (1963)

"...large changes tend to be followed by large chages -- of either sign -- and small chages by small changes"

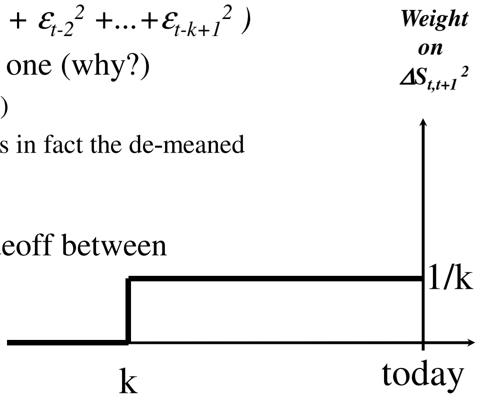


Historical STD

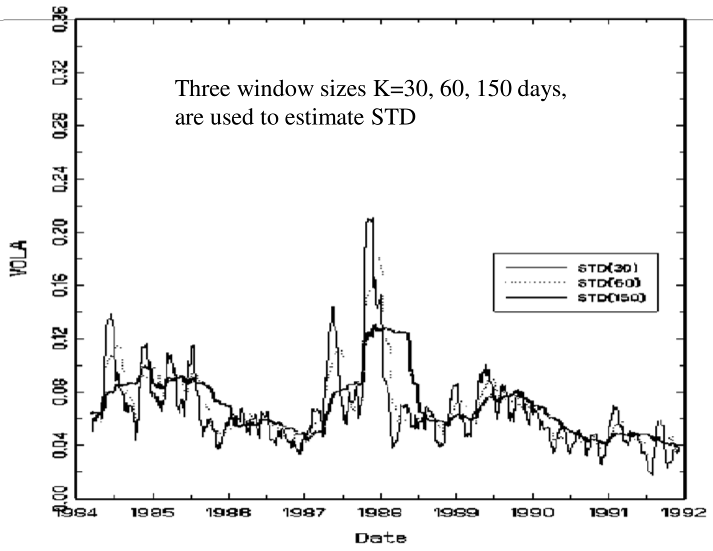
- Simple historical STDDEV is estimated by calculating the average of squared changes

$$\sigma_t^2=(1/K)(\varepsilon_t^2+\varepsilon_{t-1}^2+\varepsilon_{t-2}^2+\ldots+\varepsilon_{t-k+1}^2)$$

note that the weights sum up to one (why?)
  - » Note1: it is common to use 1/(K-1)
  - » Note2: the "suqared change" ε_{t-i}^2 is in fact the de-meaned squared returns $(\Delta S\text{-}avg(\Delta S))^2$
- The choice of K involves a tradeoff between
  - » acuuracy
  - » adaptability



Time-varying vol



OUTLINE

- **Introduction to VaR**
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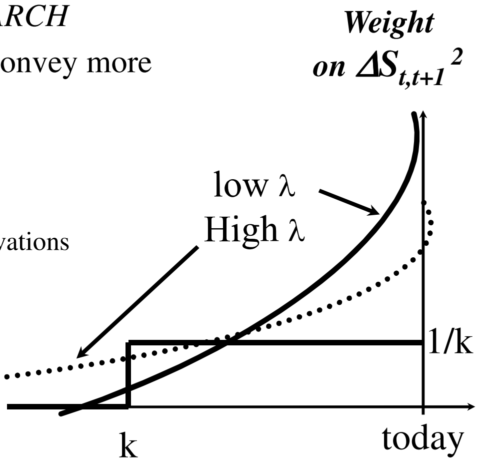
Exponential smoothing: the idea

- Examples: *RiskMetrics, GARCH*
- Idea: recent observations convey more current information
  ==> use declining weights
- **Pros:**
  - » More weight on recent observations
  - » Uses all the data
- **Cons:**
  - » Strong assumptions
  - » Many parameters to estimate



Exponential smoothing: RiskMetrics™

- Simple historical STDDEV is estimated by calculating the average of squared changes
- In RM volatility is a *weighted* average (with exp declining weights) of past changes squared:

$$\sigma_t^2=(1-\lambda)(\varepsilon_t^2+\lambda\varepsilon_{t-1}^2+\lambda^2\varepsilon_{t-2}^2+\lambda^3\varepsilon_{t-3}^2+\ldots)$$

  note that the weights sum up to one (why?)
- σ_t^2 can be also presented as

$$\sigma_t^2=(1-\lambda)\,\varepsilon_t^2+\lambda\,\sigma_{t-1}^2$$

  this is an "updating scheme" given last period's estimate of vol and the news from last period till now
- "Optimal" λ is "estimated"

Picking the "Best" Smoothing Parameter

- For each day we have σ_t and $\Delta S_{t-1,t}$.
- The "error" is $\sigma_t^2 - \Delta S_{t-1,t}^{\ 2}$
- *Mean Squared Error =* $Average[\,(\sigma_t^2-\Delta S_{t-1,t}^{\ 2})^2]$
  - » note RiskMetrics "alternative"
- We look for lambda such that is minimizes the MSE

$$\lambda^*=MIN_\lambda\{\,MSE(\lambda)\,\}$$

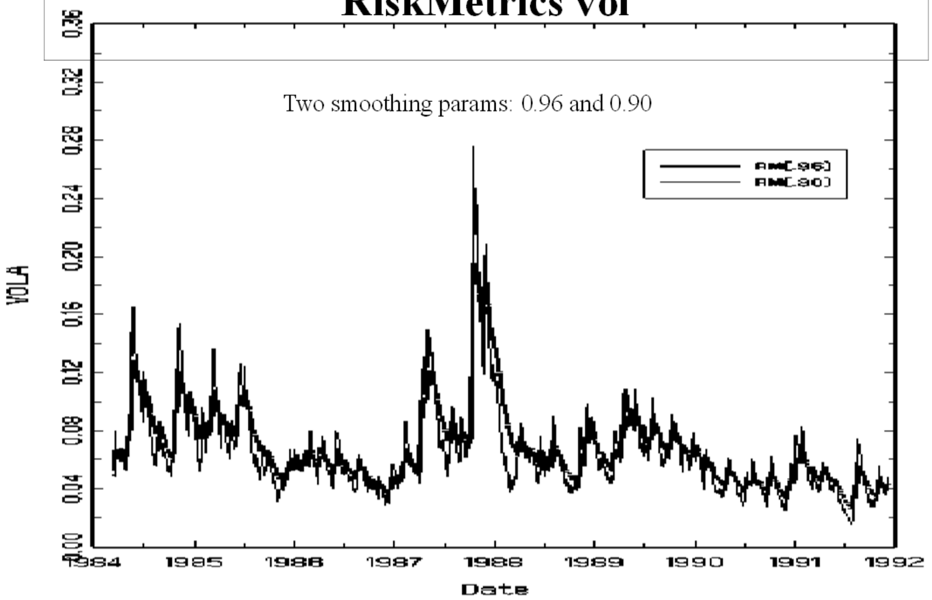
- Would λ^* for oil be the same as λ^* for interest rates?
- How do we reconcile the λ^*s for correlation
- Solution: pick λ^* to fit all assets

RiskMetrics example

$$\sigma_t^2=(1-\lambda)(\varepsilon_t^2+\lambda\varepsilon_{t-1}^2+\lambda^2\varepsilon_{t-2}^2+\lambda^3\varepsilon_{t-3}^2+\dots)$$

- Let $\lambda=0.94$
  - » *weight1* $(1-\lambda)$ $=(1-.94)$ $= 6.00\%$
  - » *weight2* $(1-\lambda)\lambda$ $=(1-.94)*.94$ $= 5.64\%$
  - » *weight3* $(1-\lambda)\lambda^2$ $=(1-.94)*.94^2$ $= 5.30\%$
  - » *weight4* $(1-\lambda)\lambda^3$ $=(1-.94)*.94^3$ $= 4.98\%$
  - » ...
  - » *weight100* $(1-\lambda)\lambda^{99}$ $=(1-.94)*.94^{99}$ $= 0.012\%$
  - » ...

RiskMetrics vol



ARCH/GARCH

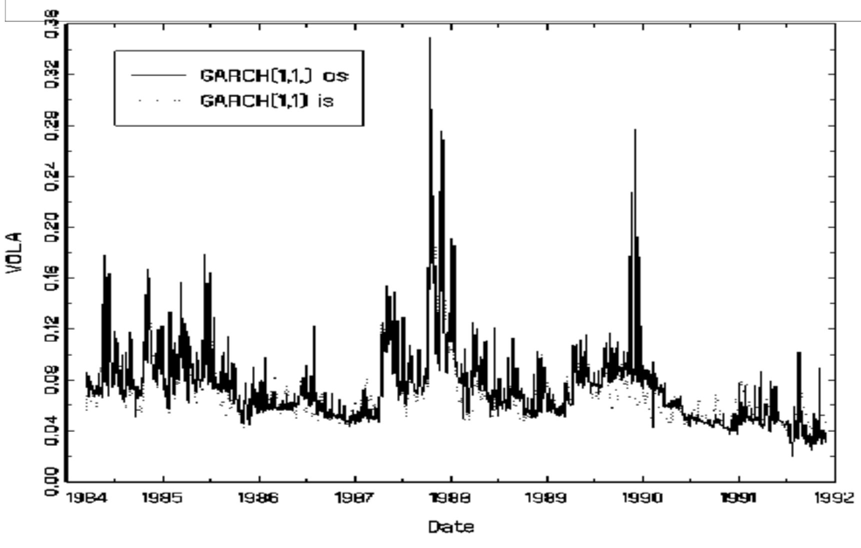
(Engle 82, Engle Bollerslev 88)

- **G**eneralized **A**uto**R**egressive **C**onditional **H**eteroskedasticity
- GARCH(1,1)

$$\sigma_t^2 = a + b\,\varepsilon_t^2 + c\,\sigma_{t-1}^2$$

- Note the close relation to riskMetrics
  (let $a=0,\ b=1-\lambda,\ c=\lambda$)
- Parameters are estimated via maximum likelihood
- GARCH, by definition, is better *in sample*
- ...but out of sample???

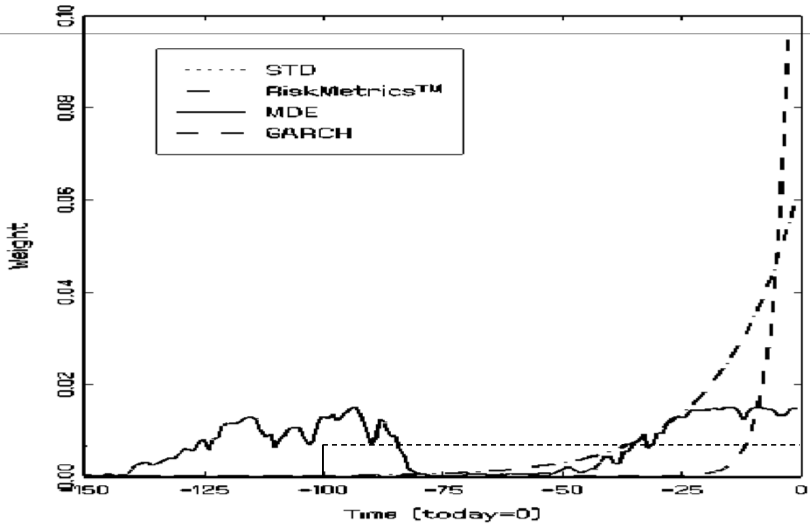
GARCH in and out of sample



Nonparametric approaches

- Examples: neural nets, density estimations,…
- **Pros:** very flexible structure
- **Cons:** data intensive ==> possibly large estimation error with limited data
- Example: estimate the changes in interest rates, CONDITIONAL on the level, the spread and vol
- ...to the extent that level and spread have information on the future path of rates, we “learn” from the past on the *conditional* distribution of interest rate changes

Weights on past ε_{t-i}^2



Historical Simulation

IN REALITY:

- Returns could be *fat-tailed* and *skewed*
- Correlations at the extremes may be misestimated

It is extremely difficult to model and estimate these effects

➔ **LET THE DATA "TELL" US**

METHODOLOGY:

- Recalculate the value of your CURRENT portfolio during the last 100 (or 250) periods
- The 5% VaR is
  - » The 5th lowest observations of the recent 100, or
  - » The 12th-13th lowest observation of the recent 250, or ...

Historical Simulation

- **Pros:**
  - » (almost) assumption-free:
    we make no distributional assumptions
  - » (almost) no parameters:
    no more vol, no more corr

  HS works in the presence of skewness, fat tails,...
- **Cons:**
  - » Very little data is used (e.g, the bi-weekly 1%VaR)
  - » Stale information lingers (long flat VaRs are typical)
  - » Extrapolation from 1-day-VaR to J-day-VaR impossible

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The *Hybrid Approach* (the best of both worlds)

- Estimates VaR by applying exponentially declining weights to the past return series
- ... and build a nonparametric time-weighted distribution

Intuition:

- » If the lowest 5 returns occurred recently (e.g., between t-1 and t-10), VaR should be higher than if they occurred long ago (e.g., t-70 and t-100)
- » If the latter is true, give these lowest returns less weight -- keep on aggregating up

The hybrid approach

- As in Historical Simulation:
  - » (+) almost assumption-free
  - » (+) OK with fat tails, skewness...
- as in EXP:
  - » (+) recent observations weigh more
  - » (+) OK for cyclical volatility
  - » (--) little data is used for low % VaRs
  - » (--) difficult to obtain *j*-period VaRs

The hybrid approach: implementation

- **Step 1:**
  - » denote by *R(t)* the realized return from *t-1* to *t*
  - » To the most recent *K* returns: *R(t), R(t-1),...,R(t-K+1),* assign a probability weight $C*1, C*\lambda, ..., C*\lambda^{K-1}$
  - » ($C=[(1-\lambda)/(1-\lambda^K)]$ ensures that the weights sum to 1)
- **Step 2:**
  - » Order the returns in ascending order
- **Step 3:**
  - » To obtain the *x%* VaR of the portfolio, start from the lowest return and accumulate weights until *x%* is reached
  - » Linear interpolation is used between adjacent points to achieve exactly *x%* of the distribution

Example

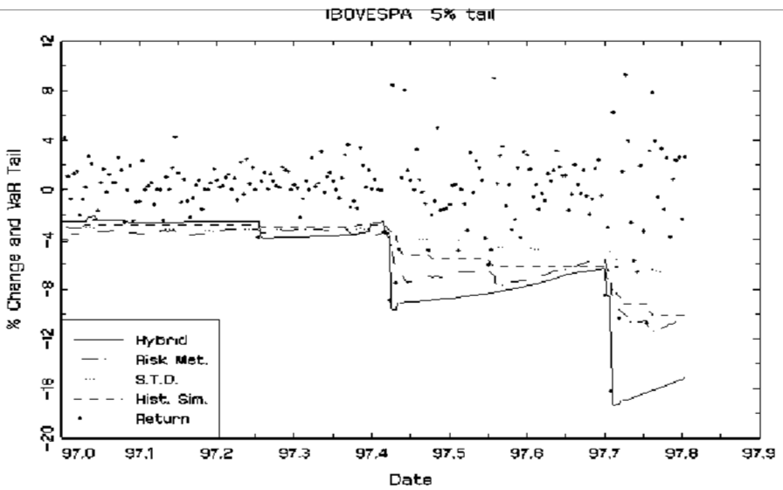
Order	Return	Periods ago	Weight	Cumul. weight	Weight	Cumul. weight
Initial Date:			Hybrid		H. S.	
1	-3.30%	3	0.0221	0.0221	0.01	0.01
2	-2.90%	2	0.0226	0.0447	0.01	0.02
3	-2.70%	65	0.0063	0.0511	0.01	0.03
4	-2.50%	45	0.0095	0.0605	0.01	0.04
5	-2.40%	5	0.0213	0.0818	0.01	0.05
6	-2.30%	30	0.0128	0.0947	0.01	0.06
25 Days Later:						
1	-3.30%	28	0.0134	0.0134	0.01	0.01
2	-2.90%	27	0.0136	0.0270	0.01	0.02
3	-2.70%	90	0.0038	0.0308	0.01	0.03
4	-2.50%	70	0.0057	0.0365	0.01	0.04
5	-2.40%	30	0.0128	0.0494	0.01	0.05
6	-2.30%	55	0.0077	0.0571	0.01	0.06

(λ=0.98, K=100)

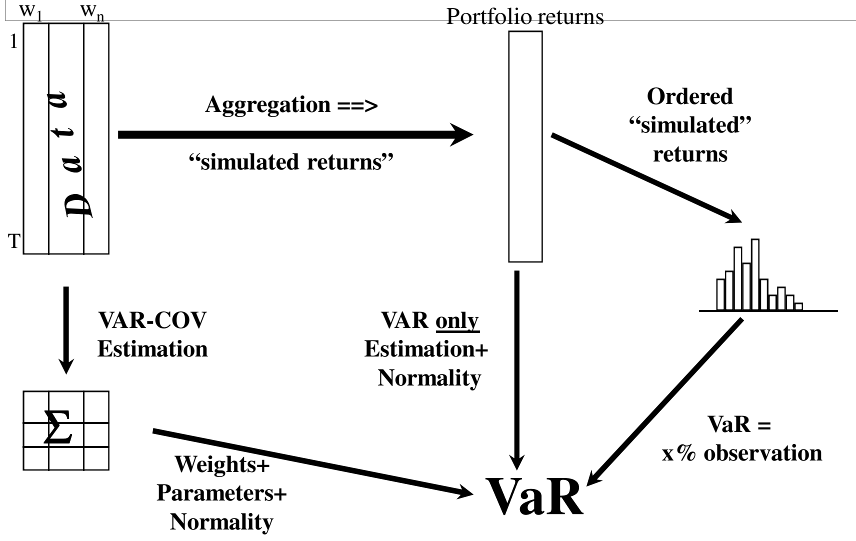
Hybrid: initial 5% VaR ==> 2.73%
25d later 5% VaR ==> 2.34%

HS (k=100d): *5% VaR* ==> 2.35%

Results: BOVESPA



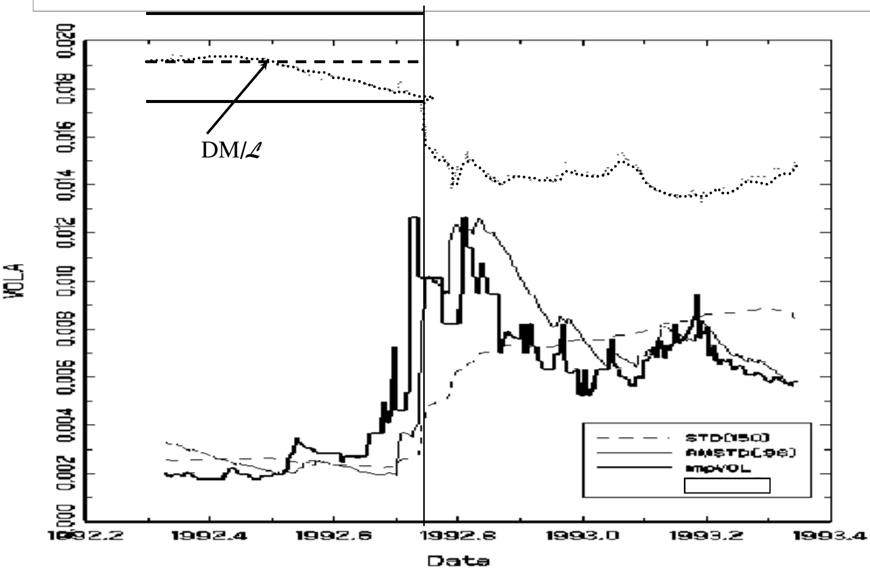
VaR and aggregation



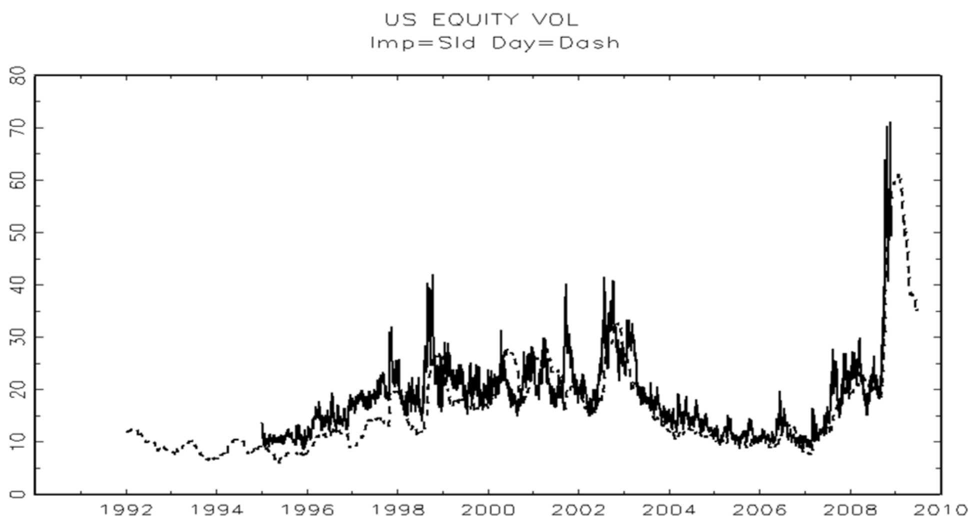
Implied vol as a vol predictor

- Pros:
  - » Uses all relevant information
  - » Completely structural
- Cons:
  - » Not available for all assets, and model is asset-specific
  - » (almost) no correlations
  - » Model (Black-Scholes, HJM, HW) may not apply
- Is the model biased?
  - » Often $\sigma_{implied} > \sigma_{realized}$
  - » There is no one "implied"
  - » Option prices compensate for crash premium, stochastic vol risk,...
  - » What do we make of $\sigma_{implied}$ as a predictor of σ ?

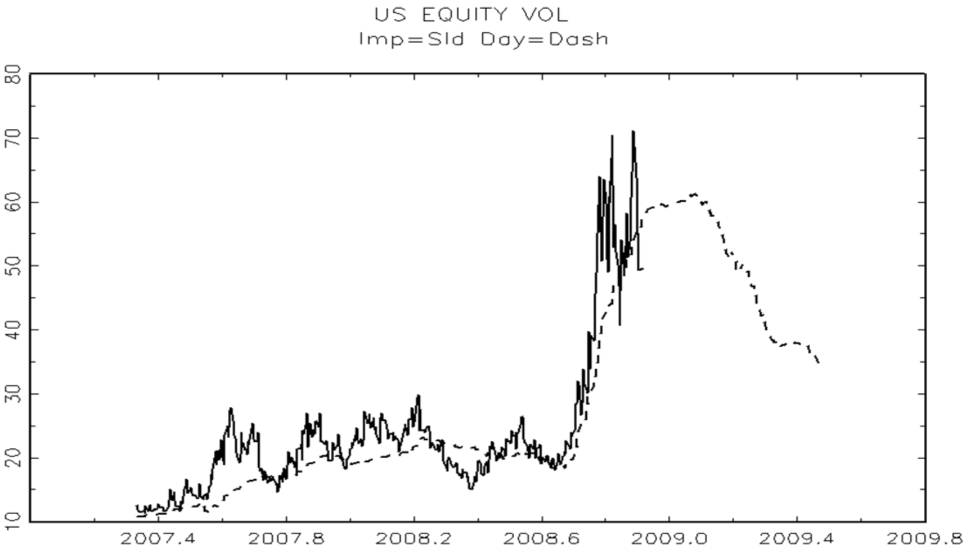
Implied vol: the GBP crash of 1992

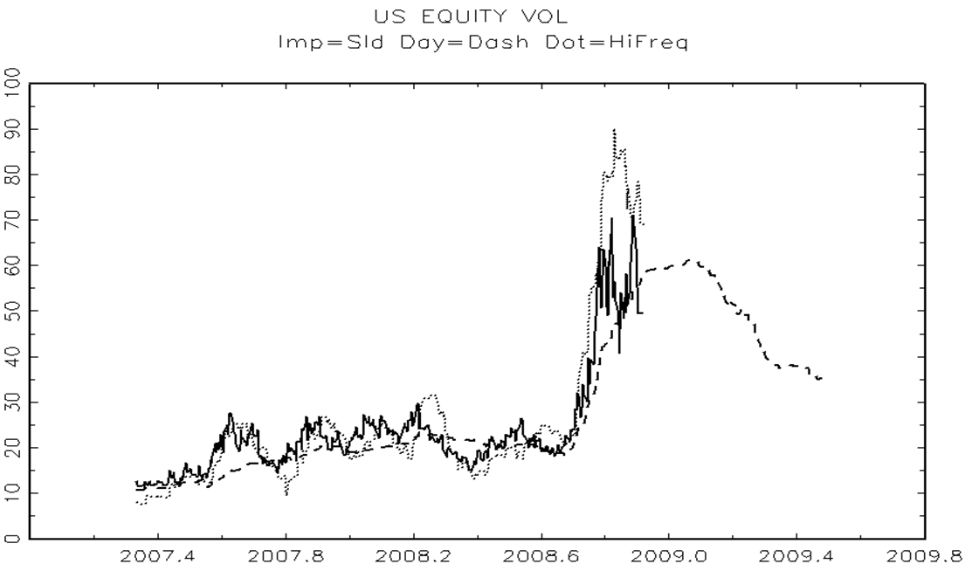


סטיית התקן הגלומה: vix לעומת היסטורית



סטיית התקן הגלומה: המשבר





מדד הפחד...

מדד הפחד – תנודות תוך-יומיות בשיא המשבר

Note not only the level, but also the intra-daily variation

Vol-of-vol peaking

What do we make of that?

מדד הפחד – הידבקות?

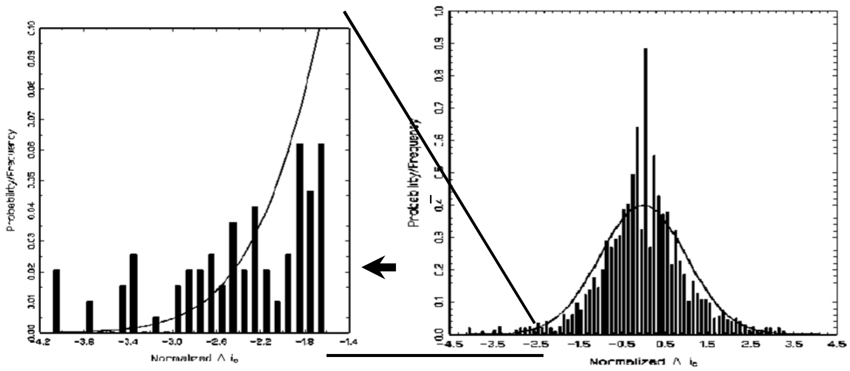
Note the recent intra-daily move

→ Must analyze *scenarios*

What do we make of that?

Normalization

- Take each IR change and divide it by its pre-estimated vol
- $\Delta i_{t,t+1} / \sigma_t$ should be distributed *N(0,1)*



Long horizon vol

- What is the *J-period CONDITIONAL variance* of $\Delta S_{t,t+J}$?
- Recall: $\Delta S_{t,t+2}=\Delta S_{t,t+1}+\Delta S_{t+1,t+2}$

  (using cont' comp' returns)

Under what assumptions do we obtain the SQRT-J rule?

J-day VaR = SQRT(J) *(1day VaR)

Recall:

$$VAR(\Delta S_{t,t+1}+\Delta S_{t+1,t+2}) = VAR(\Delta S_{t,t+1}) + VAR(\Delta S_{t+1,t+2}) + 2\,COV(\Delta S_{t,t+1},\Delta S_{t+1,t+2})$$

Long horizon vol: assumptions

$$VAR(\Delta S_{t,t+1}+\Delta S_{t+1,t+2})=VAR(\Delta S_{t,t+1})+VAR(\Delta S_{t+1,t+2})+2COV(\Delta S_{t,t+1},\Delta S_{t+1,t+2})$$

To obtain the "**SQRT-*J* rule**" we need to assume

- » *A1:* $COV(\Delta S_{t,t+1},\Delta S_{t+1,t+2})=0$
- » *A2:* $VAR(\Delta S_{t,t+1})=VAR(\Delta S_{t+1,t+2})$

- With these assumptions:

$$VAR(\Delta S_{t,t+1}+\Delta S_{t+1,t+2})=2VAR(\Delta S_{t,t+1})=2\,\sigma_t^2$$

$$==> \quad STD(\Delta S_{t,t+2})=SQRT(2)\,\sigma_t$$

...and so on for J-day returns

The empirical record of the *"SQRT(J) rule"*

The reliability of the ***"SQRT(J) rule"*** depends on the reliability of the assumptions

A1: $COV(\Delta S_{t,t+1}, \Delta S_{t+1,t+2})=0$

"no predictability", or "no mean-reversion"

A2: $VAR(\Delta S_{t,t+1})=VAR(\Delta S_{t+1,t+2})$

"constant volatility" or "no mean-reversion in volatility"

We need to determine:

- When would you expect A1 or A2 **not** to work?
- Is there a predictable bias?

No predictability assumption

A1: $COV(\Delta S_{t,t+1}, \Delta S_{t+1,t+2})=0$

- Holds true for most financial series (e.g., stock prices, FX)
- However, interest rates DO exhibit mean reversion

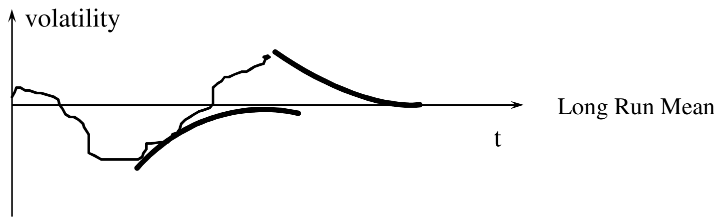
$COV(\Delta S_{t,t+1}, \Delta S_{t+1,t+2})$ <?> 0

➔ ***4*-quarter VaR <?> SQRT(*J*) *(1qtr VaR)**

Constant vol assumption

A2: $VAR(\Delta S_{t,t+1})=VAR(\Delta S_{t+1,t+2})$

- Most financial assets exhibit mean-reverting volatility



$VAR(\Delta S_{t,t+1})$<?>$VAR(\Delta S_{t+1,t+2})$

➔ ***4*-quarter VaR <?> SQRT(*J*) *(1qtr VaR)**

Mean reversion: example

- $X_{t+1}=a+bX_t+e_{t+1}$

  $STD_t(\Delta X_{t,t+1})= STD_t(a+bX_t+e_{t+1} - X_t)= \sigma_t$
- SPSE b=0.9, σ_t=10%

  ==> $STD_t(\Delta X_{t,t+1})=10\%$
- $\Delta X_{t,t+2}=$ ... *(write X_{t+2} in terms of X_{t+1}, then in terms of X_t)*

  $VAR_t(\Delta X_{t,t+2})=(1+b^2)\sigma_t^2=(1+0.81)*(10\%)^2$

  ==> $STD_t(\Delta X_{t,t+2})=1.34*(10\%)$
- Lower than the SQRT-J rule volatility:

  1.41* (10%)
- Especially relevant with short term arbitrage strategies

Benchmarking & backtesting VaR: Methodology

By definition, at any given period the following must hold:

$$Prob[R(t+1)<-VaR(t)]=x\%$$

Benchmarking and backtesting is done by observing
the properties of the frequency and size of VaR violations

Define: *I(t)=1* if the *VaR(t)* is exceeded, 0 otherwise

Attributes:

- Unbiasedness:
  - » Unconditional: *avg[I(t)]=x%*
  - » Conditional: low *Mean Absolute Error*
- Proper Updating: *I(t) should be i.i.d..*
  ==> *Autocorr[I(t)] = 0*

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The VaR of derivatives: introduction

- A derivative is priced off of an underlying asset
  → Changes in the value of a derivatives are derived from changes in the underlying (and the "factor(s)" moving the underlying)
- **Linear derivatives**: Δderivative is linear in Δfactor(s)
  - $P(f) = \alpha + Delta * f$ → $\Delta P = Delta * \Delta f$
  - forwards, futures, swaps
- **NON-linear derivatives**: ΔP is Nonlinear in Δfactor(s)
  - $P(f) = \alpha + Delta(\mathbf{X}_t) * f$ → $\Delta P = Delta(\mathbf{X}_t)\ \Delta f$,
    where X_t is a state dependent variable
    (e.g., the level of interest rates, the "moneyness" of the option)
  - options, MBSs, Bradys, caps/floors

How to calculate the VaR of derivatives?

- If linear: straightforward
  $P = \alpha + Delta * f$
  → $\Delta P = Delta * \Delta f$
  → $VaR_P = \mathbf{Delta} * VaR_F$
- Every asset is LOCALLY linear
- …but for large moves (long horizon VaRs, stress scenarios,...) nonlinearity matters
- Two methods/approaches to address nonlinearity:
  - Full re-valuation
    (usually in conjunction with *structured Monte Carlo*)
  - Approximation to the nonlinearity effect
    (*"the Greeks"* using Taylor expansion)

Linear derivatives: the VaR of FX forwards

- FX forward contract: exchange \$F for DM1 in at *t+T*
- Forwards are priced via covered parity: $F_{t,T} = S_t (I_{t,T}/I^*_{t,T})$
- It is derived by arbitrage, using the fact that the following are equivalent:
  - » purchase DM forward
  - » short \$bond at $I_{t,T}$, convert into DM, long DMbond at $I^*_{t,}$
- In log terms $\Delta F = \Delta S + \Delta I - \Delta I^*$
  - » in words: the change in the value of a forward contract is equivalent to (by arbitrage) the change in the spot rate, plus the change in the \$ bond, less the change in the DM bond

==> The VaR of the forward depends linearly on the *vol* and *corr* across the three variables: [ΔS_t , Δ\$bond , ΔDMbond]

The VaR of an FX forward: example

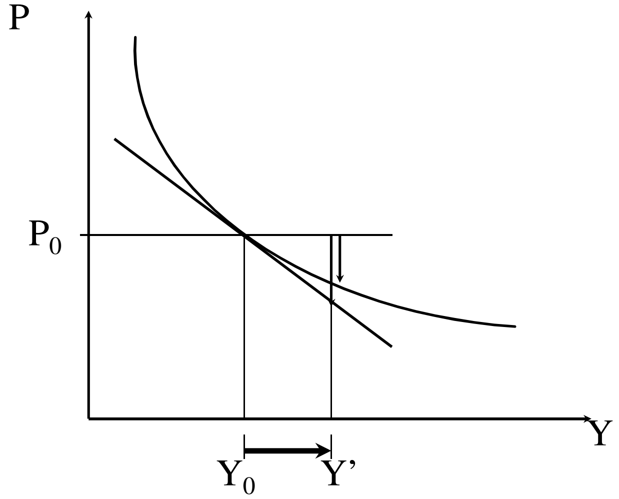
- Recall: $\Delta F = \Delta S_t + \Delta I - \Delta I^*$
- Hence: $\sigma^2_{\Delta F} = \sigma^2_{\Delta S} + \sigma^2_{\Delta I} + \sigma^2_{\Delta I^*}$
  $+2cov(\Delta S,\Delta I)-2cov(\Delta S,\Delta I^*)-2cov(\Delta I, \Delta I^*)$
- Example:
  - » S=\$0.555/z, I=5%, I*=3%, T=1yr ==> F=\$.566/z
  - » Notional amount z1.8MM=\$1MM
  - » Suppose $\sigma_{\Delta S}$=70bp/day, $\sigma^2_{\Delta I}=\sigma^2_{\Delta I^*}$=7bp/day, CORR=0
  - » **$VaR_{\Delta S}$= \$7000*1.65=11,550**,
  - » $VaR_{\Delta I}=VaR_{\Delta I^*}$= 700*1.65=1,155
  - » **$VaR_{\Delta F}$ = sqrt(11,550 ^{2} + 1,155 ^{2} + 1,155 ^{2}) =\$11,664**
  - » Why is $VaR_{\Delta F}$ so close to $VaR_{\Delta S}$?

Nonlinear derivatives

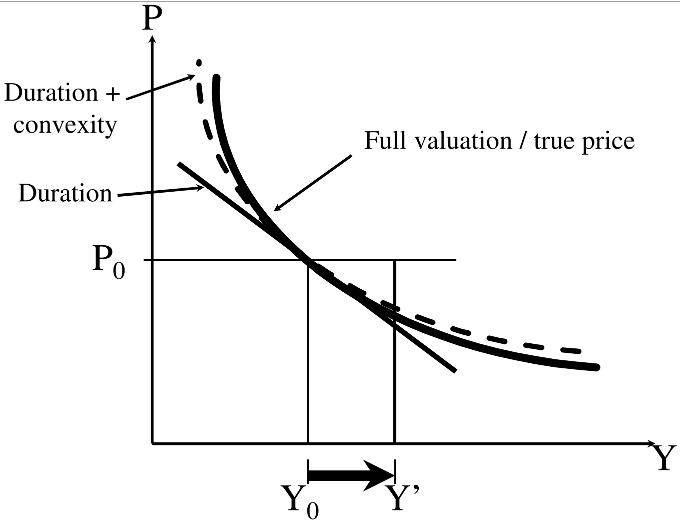
- *Recall linear derivatives: $\Delta P = Delta \ \Delta F$*
- In the case of nonlinear derivatives, the DELTA is state dependent:

  $$\Delta P = Delta(X_t) * \Delta F$$
- Examples (by increasing complexity):
  - » Bond are nonlinear in interest rates
  - » Options are nonlinear in the underlying
  - » Convertibles are nonlinear in the underlying
  - » Callable & convertible bonds are nonlinear in the underlying and in interest rates
  - » Defaultable (e.g., Brady) bonds are nonlinear in the default probabilities
  - » MBSs are nonlinear in interest rates

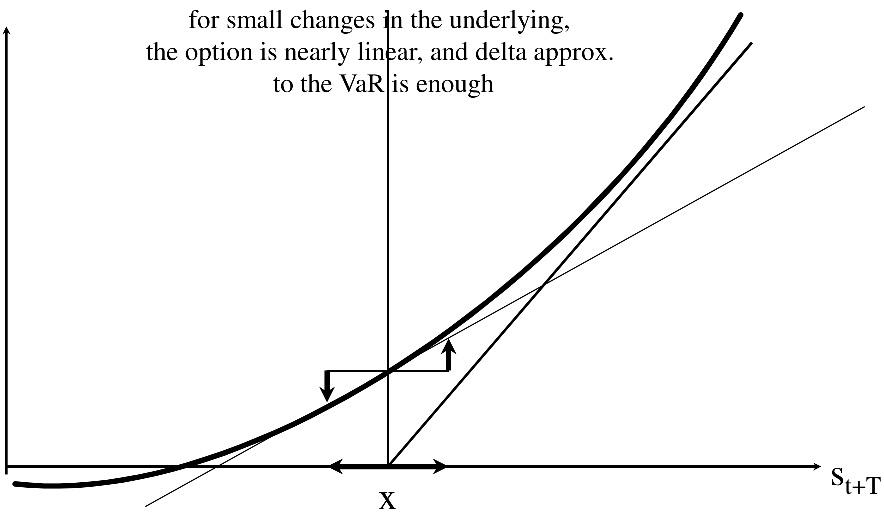
The problem with duration



Duration + convexity



The VaR of options: small moves



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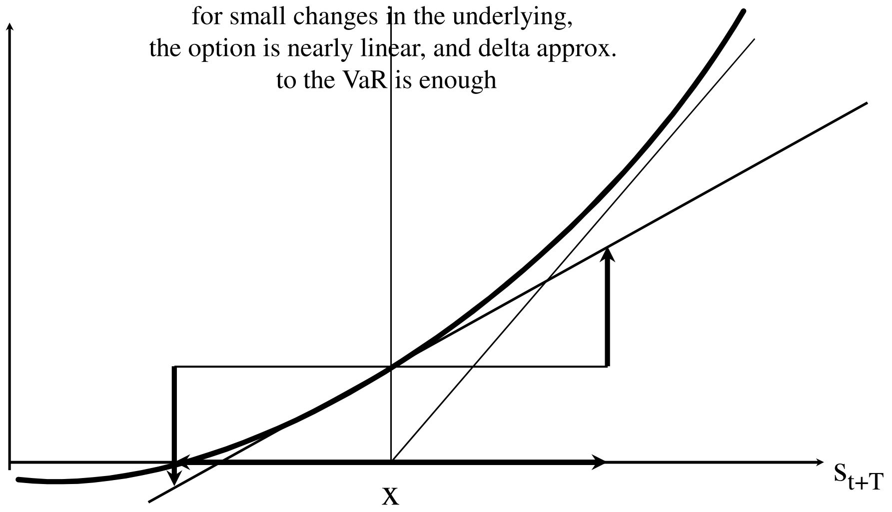
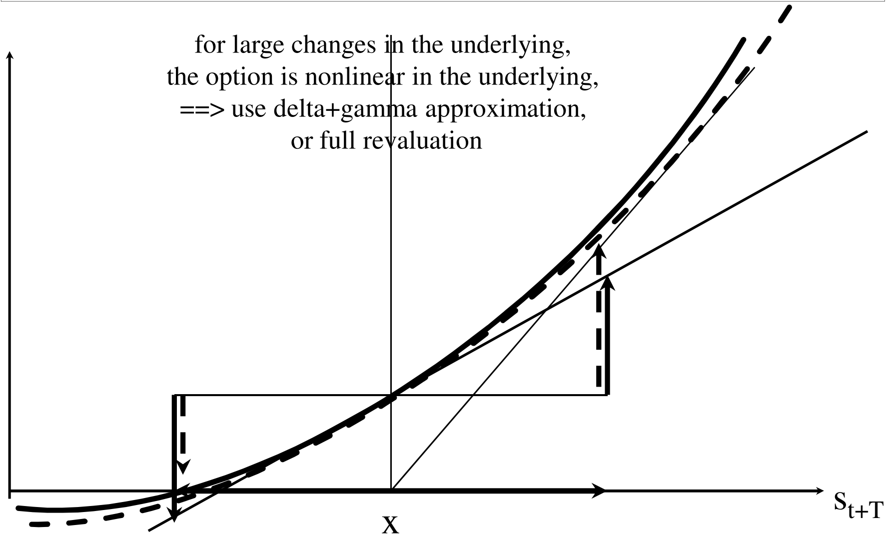
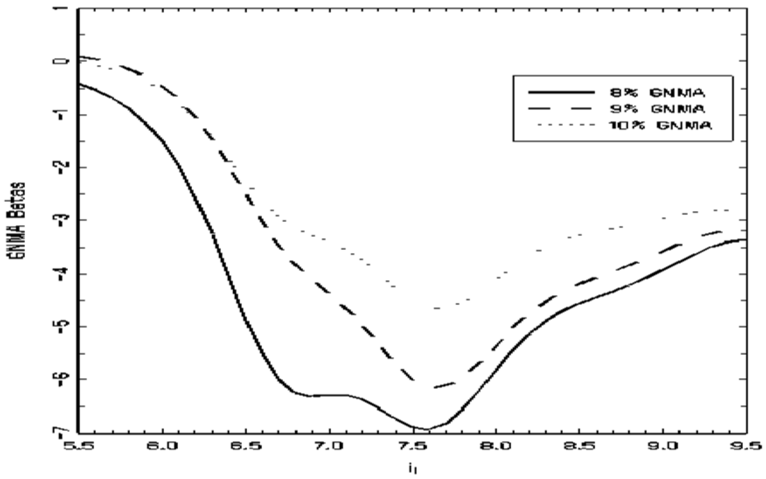


The VaR of options: convexity correction

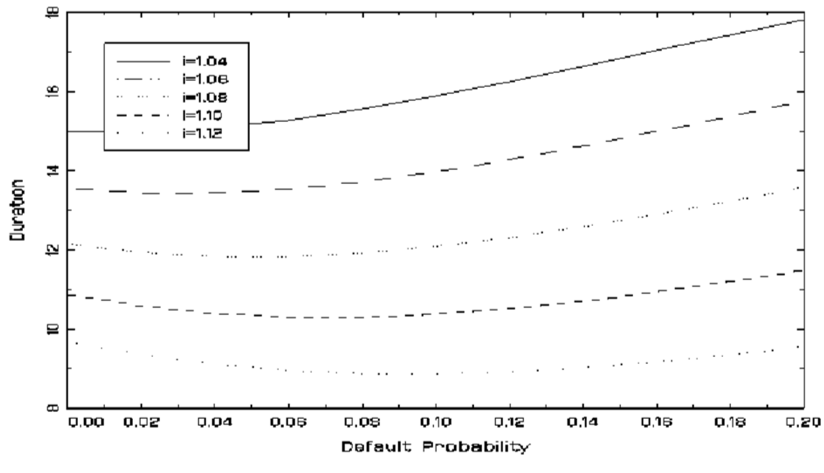


Empirical interest rate sensitivity of MBSs



Source: "Pricing of Mortgage-Backed Securities in a Multifactor Interest Rate Environment: A Multivariate Density Estimation Approach", Boudoukh, Richardson, Stanton and Whitelaw, ***Review of Financial Studies 1996***

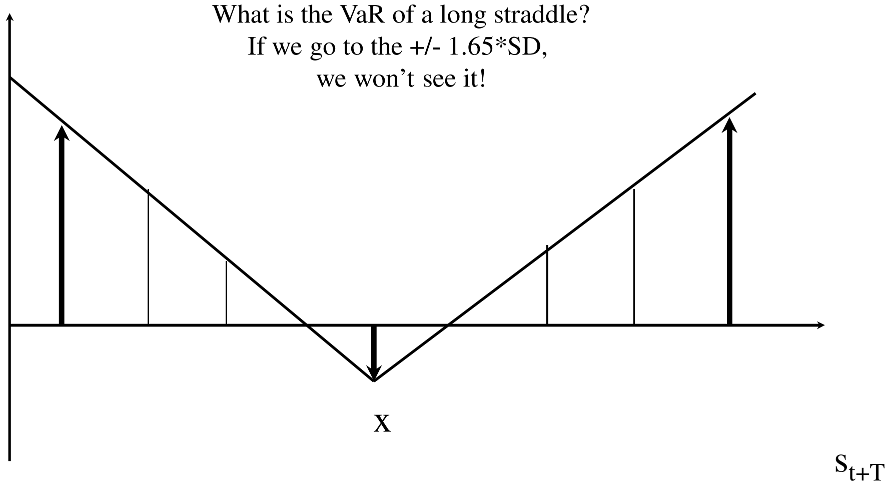
Theoretical interest rate sensitivity of Bradys



Source: "Hedging the Interest Rate Risk of Brady Bonds" , Ahn, Boudoukh, Richardson and Whitelaw

The VaR of options: a straddle

What is the VaR of a long straddle?
If we go to the +/- 1.65*SD,
we won't see it!



The VaR of a portfolio of derivatives

- As the previous example shows, the problem is not only nonlinearities, but also ***nonmonotonicities*** (17 letters)
- The problem with full the revaluation approach is its computational cost/time
  - » we need to cover the entire range of the distribution ==> simulation
  - » with N state variables (interest rates, exchange rates, default spreads, etc), we need to revalue the portfolio thousands of times.

    e.g., revalue MBSs, caps, swaptions etc for 10,000 scenarios
  - » solution: reduce the number of states

    e.g., the level and spread (= 2 factors) may suffice to describe the entire term structure

OUTLINE

- **Introduction to VaR**
  - » Statistical framework. Risk and diversification: some examples. Possible applications. Visual interpretation.
- **The Stochastic Behavior of Asset Returns**
  - » Time variations in volatility. VaR: approaches and comparison. The *Hybrid Approach* to VaR.
- **Beyond Volatility Forecasting**
  - » The VaR of derivatives and interest rate VaR. **Structured Monte Carlo.** Extreme events and correlation breakdown. Stress testing and scenario analysis. Worst case scenario

Structured Monte Carlo: basic intuition

- Generating scenarios for **one** variable which is $N(\mu, \sigma^2)$
  - » generate 10,000 simulations of $N(0,1)$. Denote: $Z_1, \ldots Z_{10,000}$
  - » calculate scenarios $S_{t+1,i} = S_t * exp\{\mu + \sigma * Z_i\}$
  - » Revalue the derivative for each $S_{t+1,i}$

- Generating scenarios for **K** variables which are $N(\underline{M},\underline{\Sigma})$
  (where $\underline{M}$ is a vector of length K, and $\underline{\Sigma}$ is a K by K matrix)
  - » generate 10,000 K-vectors $N(0,\underline{I_K})$'s: $\underline{Z}_1, \ldots \underline{Z}_{10,000}$
    (where $\underline{I}_K$ is K by K unit matrix)
  - » calculate scenarios $\underline{S}_{t+1,i} = \underline{S}_t * exp\{\underline{M} + \underline{A}' \underline{Z}_i\}$
    (where $\underline{A}$ is the "square root matrix" of $\underline{\Sigma}$, namely $\underline{A}'\underline{A} = \underline{\Sigma}$)
  - » Revalue the derivatives for each $\underline{S}_{t+1,i}$ set of values

Structured Monte Carlo: discussion

- <u>The main advantage</u>: **correlated scenarios**
- Compare to independent scenario analysis
  - » a 200bp shift in interest rates, a 25% decline in equities, a 500bp increase in the Brady strip spread,...
  - » what do we make of such isolated scenarios? do they make <u>economic</u> sense?
- <u>The main disadvantage</u>: **correlation breakdown**
  - » what happens to global yield correlation during an oil crisis?
  - » what happens to strip spreads during an EM currency crisis?
  - » what happens to corporate--equity correlation during an equity crisis?

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Generating scenarios: what is "reasonable stress"?

Odds in 100,000

SDs	Normal dist'n	S&P 500	Yen/$	10yr Rate
2	4500	3700	5600	5700
3	270	790	1300	1300
4	6.4	440	310	240
5	**0**	**280**	**78**	**79**
6	0	200	0	0

Correlation breakdown

Event	Date	Variables	Prior to	During
ERM	Sep92	GBP/$, GBP LIBOR	-0.10	0.75
Mexico	Dec94	Peso/$, 1mo Cetes	0.30	0.80
87 crash	Oct87	Junk yield , 10yr Treasury	0.80	-0.70
Iraq	Aug90	10yr JGBs , 10yr Treasury	0.20	0.80
Asian Crisis	1997/8	Brady debt of Bulgaria and the Philippines	0.04	0.84

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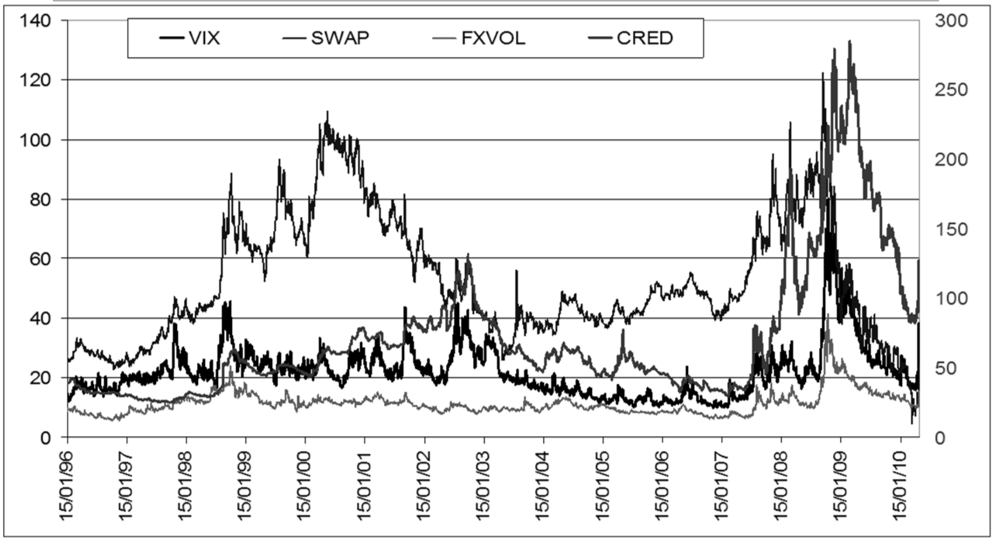
Generating stress scenarios in practice

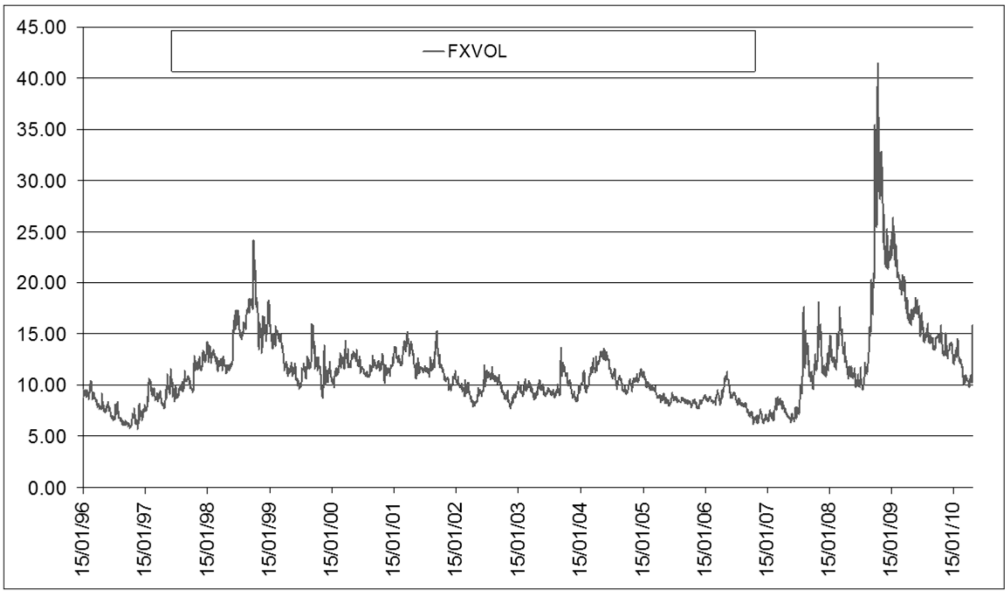
- Common practice: examine historical events
- Links to the historical simulation methodology:
  - » use HS to generate the empirical distribution for the current portfolio
  - » use HS to examine the "five worst weeks" given the current portfolio, when did they occur, and what were the circumstances
- Remember: there is no way to apply common statistical techniques, with so few (and economically different) data points
- However: extreme value theory is now commonly applied to the problem
- Its usefulness is very questionable

Link: ***tech ticker Roubini Bank Stress Tests: Design Is Fine, But Concept Fundamentally Flawed, Roubini Says***

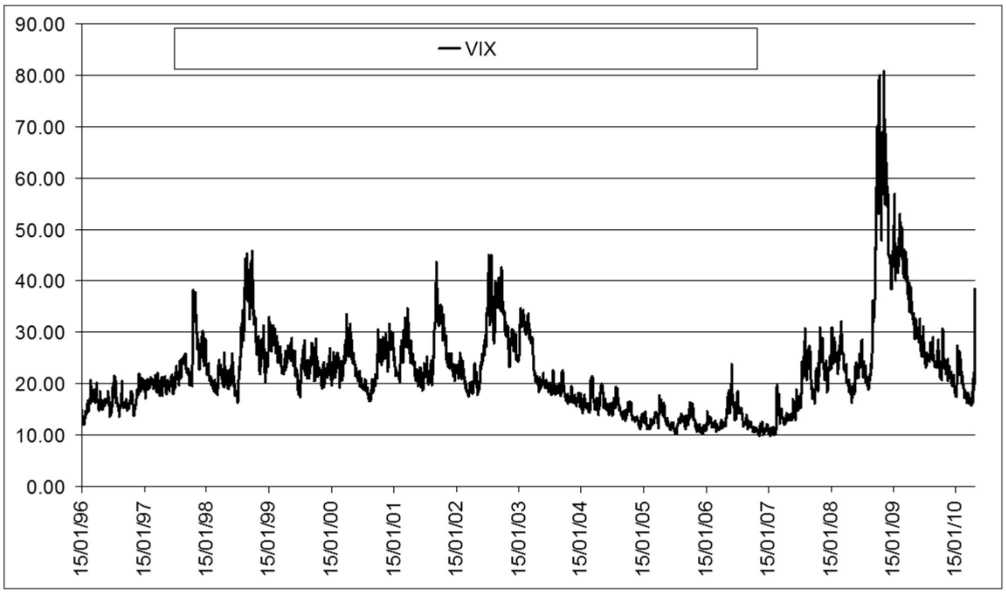
מדדי סיכון



RM 117



RM 118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70.00
60.00
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20.00
10.00
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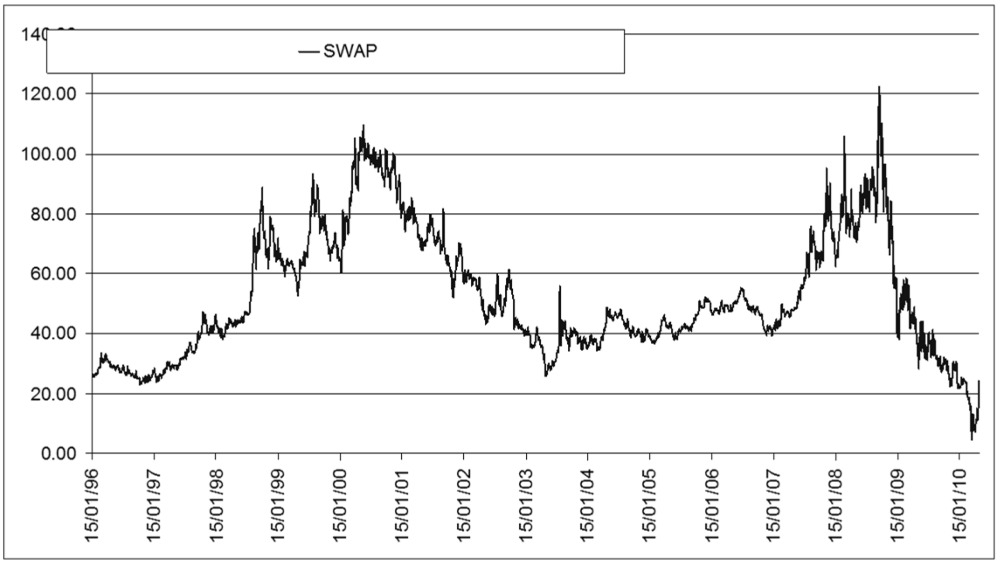
RM 119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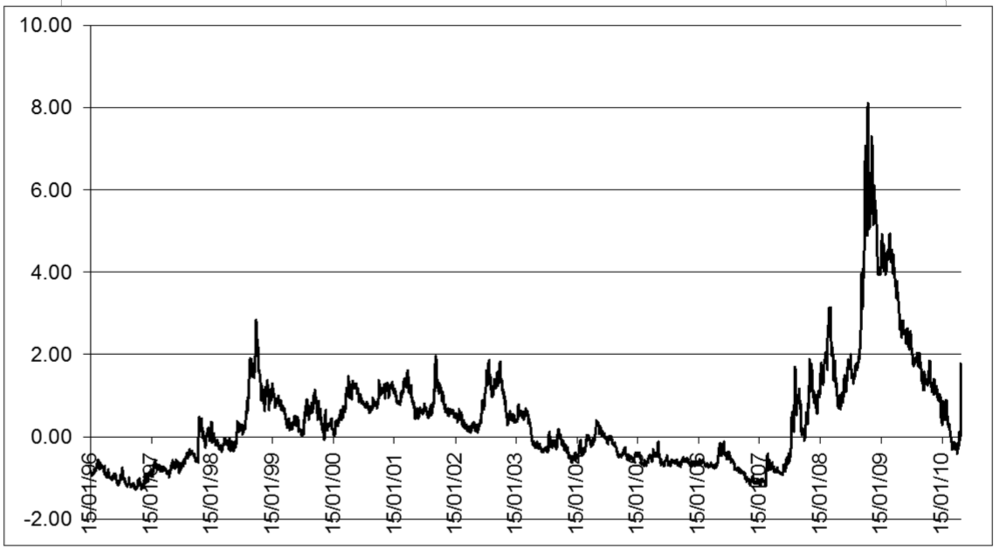
15/01/08
15/01/09
15/01/10

RM 120

ברומטר סיכון מנורמל לטרום המשבר



Sample Problems

The annual standard deviation of the Japanese stock market (in Yen) is 20%, and that of the Yen/$ exchange rate is 10% per annum. You believe that the expected return on the Japanese market (in Yen) is 10% per annum, and the expected rate of appreciation of the Yen over the coming year is 5%.

Assumptions:

VaR is the 5th percentile (1.65 standard deviations),

The correlation of the Yen/$ rate with the Japanese market is zero

Questions:

a. Suppose you invest $1,000,000 in the Japanese market (currency unhedged). What is your annualized 5% Value at Risk (in $ terms)?

b. How would you implement a currency hedge on this equity investment? Is this a perfect hedge? How could you improve the hedge?

c. What is the effect of fully hedging the currency exposure (i.e., by how much would you reduce your $ exposure)?

d. Suppose the $ risk free rate is 4%. What is the Sharpe Ratio of a currency-hedged and currency-unhedged investment in the Japanese market?

e. Based on your calculations, which investment strategy should you prefer? Explain why the results are economically counterintuitive, and how do they result from the assumptions?

SOLUTION

$VaR=1,000,000*1.65*sqrt(.2^2+.1^2)=**$368,951**

some may have calculate around the expected return of x%, which is OK ($426,138)

Via Nikkei futures, traded/readjusted monthly for example, hedging the underlying position 1:1. This is not a perfect hedge due to the residual amount during the month, hence increasing the frequency of readjustment of hedge will improve it. Also, a statistical regression can provide a hedge ratio more precise than the 1:1 myopic hedge.

(c) YenVaR=1,000,000*1.65*.20=**$330,000**.

Hence 368,951-330,000=**$38,951**

(d) E [excess unhedged return] = (1.10*1.05-1.04)*100 = 11.5%

E [excess hedged return] = (1.10-1.04)*100 = 6%

and SD[excess unhedged return] = sqrt(.2^2+.1^2)*100 = 22.36%

SD[excess hedged return] = sqrt(.2^2)*100 = 20%

==> Sharpe[unhedged] = 11.5/22.36 = **0.51**

Sharpe[hedged] = 6/20 = **0.30**

(e) You should prefer to remain currency unhedged, particularly due to your belief that the Yen is going to appreciate. This assumption is economically implausible, since the inflation differential is small.

True/False, multiple choice etc., with a brief explanation

(a) In comparing the volatility forecasts from a simple standard deviation model with a lookback period of 100 periods (STD(100)) to that of 50 periods (STD(50)), you would expect

(1) The STD(50) to be more biased on average than the STD(100) relative to the true volatility

(2) The STD(50) to be more volatile forecast series than the STD(100)

(3) Both (1) and (2)

(4) None of the above

(b) (True/False) If the volatility of an asset moves around very slowly, you would expect an exponential smoothing parameter closer to one (e.g., 0.97) to be a better vol forecasting parameter than a lower smoothing parameter (e.g., 0.94)

(c) If asset returns are more "fat-tailed" than what conditional normality indicates, the VaR is understated

(d) If current volatility is above its long run mean, then the square root rule for long horizon volatility will understate the true long horizon volatility.

(a) Solution: (2), due to sampling variation or true vol variation. There is no question of bias, though.

(b) Solution: True. A higher smoothing parameter will reduce the sampling variation, and will miss little in the way of current information, relative to the low smoothing parameter.

(c) Solution: True. The x% tail of returns will be larger than the theoretical one.

(d) Solution: False. The exact opposite is true, namely, since volatility is expected to decline, the square root rule will overstate long horizon volatility.