The Pricing of Stripped Mortgage-Backed Securities

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Abstract

Interest only (IO) and principal only (PO) stripped mortgage-backed securities (MBS) are derivative securities which pay out only the interest component (IO) or principal component (PO) of the cash flows from the underlying mortgages. Their often extreme and nonlinear response to interest rate movements has led to many dramatic losses by traders in these securities, and makes it vital to specify correctly any model used to value or hedge them. This paper uses a large sample of market prices of IO and PO securities, with a wide range of coupon rates over the period June 1990–January 1995 to understand the main properties of IO/PO prices, and to determine the important features that any realistic prepayment and valuation model must capture. We fit a rational mortgage valuation model to the data, and use nonparametric techniques to explore the model's ability to match the behavior of both prices and prepayment over the sample period, leading to several potentially fruitful directions for future research.

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1 Introduction

A significant fraction of the enormous mortgage-backed security market[1] consists of mortgage derivatives, whose payoffs are functions of the payoffs of an underlying mortgage-backed security (MBS) or pool of MBS. The main mortgage derivatives are Stripped Mortgage-Backed Securities (SMBS) and Collateralized Mortgage Obligations (CMO).

As the market for MBS and mortgage derivatives has developed, there have been many well-publicized cases of large losses incurred by supposedly sophisticated investors in these securities. For example, Merrill Lynch lost approximately $250 million on a single PO transaction in 1987. In 1993, funds managed by Hyperion Capital Management (founded by Lewis Ranieri, often regarded as the "father of the mortgage-backed securities market") sustained disastrous losses on their holdings of IO securities as interest rates fell. The complex payoff structures of these securities serve to magnify the problems inherent in pricing any MBS. The key factors underlying both mortgage and mortgage derivative prices are interest rates and mortgage holder prepayment behavior. The level of interest rates determines the present value of the future cash flows from the securities, and prepayment affects both their level and timing. High prepayment rates typically increase PO prices, all else being equal, since PO holders receive their payments earlier than they would otherwise. On the other hand, high prepayment rates decrease IO values, since IO holders receive none of the principal, and their interest payments stop immediately after prepayment.

To price and hedge mortgages and MBS requires a model for interest rate dynamics, and for the prepayment behavior of mortgage holders in response to changes in interest rates (and possibly other factors). Mortgage holders possess an option to prepay their existing mortgage and refinance their property. They are more likely to do so as interest rates, and hence refinancing rates, decline further below the rate of their current mortgage. Thus a mortgage with an $X\%$ coupon is roughly equivalent to a default-free $X\%$ coupon-bearing bond and a short position in a call option on that bond, with an exercise price of par. This option component induces a concave relation between the value of a mortgage and the level of interest rates (the so called "negative convexity" of mortgages). Early academic research, such as Dunn and McConnell [7, 8], treated mortgages exactly as a portfolio of

[1] The total face value of outstanding mortgage-backed securities of all types was over $2 trillion in 1994.

bond plus option, setting up and solving a valuation equation for the value of the mortgage, and determining the optimal exercise strategy of frictionless mortgage holders as the solution to a free boundary problem. Though internally consistent, such models produced two rather unfortunate results. First, all mortgage holders find it optimal to refinance at the same time, so there will be no prepayment (or some "background" level of prepayment) until one instant when all remaining mortgages in a pool will suddenly prepay. Second, mortgage prices can never exceed par.

In response to these shortcomings, a second strand of research emerged (examples include Schwartz and Torous [20] and most Wall Street models), in which mortgage prepayment is modeled as a function of some set of (non-model based) explanatory variables, and the resulting "prepayment function" is inserted into a Monte Carlo simulation algorithm to perform the valuation. Most such models use either past prepayment rates or some other endogenous variable, such as burnout,[2] to "explain" current prepayment. Their use of large numbers of explanatory variables, including lagged dependent variables, combined with a lack of any theoretical restrictions on the nature of the relationship, makes such models very good at predicting prepayment a short time into the future. However, these same characteristics make the models prone to finding spurious relationships between variables. In addition, since these models are really heuristic reduced form representations for some true underlying process, it is impossible to know how they would change in response to a shift in the underlying economy, such as a change in interest rate volatility, or a reduction in the costs of refinancing. All we know is that there would be *some* change.

Recently, the old rational approach to mortgage valuation has been resurrected. To allow mortgage prices to exceed par, Timmis [23], Dunn and Spatt [9], and Johnston and Van Drunen [13] add transaction costs that must be paid by mortgage holders on refinancing. Stanton [22] further extends these models, producing mortgage prepayment behavior that can exhibit most of the features noted in the data, such as

1. Burnout.

[2] Burnout refers to the dependence of expected prepayment rates on cumulative historical prepayment levels. The higher the fraction of the pool that has already prepaid, the less likely are those remaining in the pool to prepay at any interest rate level. See, for example, Richard and Roll [19].

2. Some mortgages are prepaid even when their coupon rate is below current mortgage rates.

3. Some mortgages are not prepaid even when their coupon rate is above current mortgage rates.

Since this model describes the prepayment *process* of mortgage holders, rather than the outcome of this process (as do the empirical models), it is robust to changes in the underlying economy. McConnell and Singh [15, 16] have recently used this model to price CMOs, but its pricing implications have not yet been thoroughly investigated, owing at least in part to a lack of reliable data.[3] This paper fits the parameters of the model using a large sample of market prices of specific IO and PO securities, with a wide range of coupon rates, over the period June 1990–January 1995. Using nonparametric techniques, we analyze the model's pricing and prepayment errors. This allows us to develop an understanding of the main properties of IO/PO prices and mortgage prepayment behavior, and to determine the important features that any realistic prepayment and valuation model must capture if it is to have any hope of being a useful tool, for valuation and hedging. Our results indicate several directions in which the valuation model needs to be extended.

2 Stripped Mortgage-Backed Securities

Stripped MBS were first introduced in 1986 by the Federal National Mortgage Association (FNMA), which remains the dominant issuer.[4] Just as with regular MBS, stripped mortgage-backed security (SMBS) holders receive a fraction of the interest and principal payments made by some underlying pool of mortgages. The difference is that the interest and principal fractions *differ*. The first SMBS were *synthetic coupon pass-through securities.* For example, given a pool of 11% mortgages, synthetic 14% and 8% stripped mortgage-backed securities can be created by forming two new securities, each of which receives 1/2 of the principal payments from the 11% security, but where the interest payments are split in the ratio 7:4.

[3] Most available prices are generic TBA (To Be Announced) prices, rather than the pool-specific prices that are produced by the model.

[4] The discussion here borrows from the more detailed treatment in Hayre and Mustafa [10].

These are *not* equivalent to standard 14% and 8% MBS, since their value depends on the prepayment behavior of the underlying holders of the 11% mortgages, which will differ from that of 14% or 8% mortgage holders. The most common type of SMBS is the interest only (IO) and principal only (PO) stripped MBS, first issued in 1987. As their name suggests, holders of these securities receive a share of only the interest component (IO) or principal component (PO) of the cash flows from the underlying mortgages. Holders of regular FNMA mortgage-backed securities submit these securities to FNMA, which consolidates them into one "Megapool Trust". As with the loans underlying regular MBS, the securities in a given trust must be reasonably homogeneous. They must all be of the same loan type, and within a certain range of WAC[5] and WAM[6] values. FNMA returns to the original security holder two SMBS Trust certificates, giving the holder rights to a specific proportion of the principal and interest payments from the FNMA Megapool Trust.

2.1 Data

For the purposes of this study, a major Wall Street firm provided us with daily[7] bid prices for FNMA 6.5% to 10.5% IO and PO securities, over the period 1990–1995. The prices are of the "benchmark" trust, the most liquid trust in the opinion of the firm's traders, and the trust from which other securities are priced. For each coupon rate, the benchmark may change over the course of the sample period.[8] In addition to knowing the prices, we also know the trust numbers, which allows us to obtain additional information such as historical prepayment information for each trust from Bloomberg Financial Markets.

With respect to interest rate series, daily data for the 1990–1995 period were collected for the yield on a constant maturity 10-year Treasury security, and on a 3-month Treasury bill from Datastream.

[5]Weighted Average Coupon.

[6]Weighted Average Maturity.

[7]The data are less frequent in the early part of the sample.

[8]While some coupons have the same benchmark throughout the sample period, the benchmark changes as often as four times for one of the FNMA coupons.

2.2 Characteristics of Interest Rates and Security Prices

Figures (1)–(3) show the behavior of 6.5%, 8.5% and 10.5% IO and PO securities over the sample period. In each figure, the first part is a time series plot of the prices, shown together with the 10-year Treasury yield over the same period. The second part is a scatter plot of the asset prices against the 10-year Treasury yield. In the second part of figure (2), the two symbols (square/circle) represent two different benchmark trusts. Looking first at the interest rate series, we see two main features. First, the sample period was characterized by significant changes in interest rates (the range is from just over 5% to 9%), and also multiple periods at similar rates. For example, rates were around 7% in both early 1992 and the middle of 1994. The range of rates is encouraging for estimation of any model, and the existence of multiple periods at similar rates allows the study of features such as seasoning and burnout.

Each of the coupons display the same basic features. Both IO and PO prices vary substantially over the sample period in response to interest rate changes. There is a pronounced negative relationship (positive duration) between PO prices and the level of interest rates. The total (nondiscounted) cash flow received by PO holders equals the principal on the underlying loan. As interest rates move, both the timing of these cash flows and the discount rate affect the asset value in the same direction. As rates fall, prepayment increases, so the cash flows are received earlier, increasing their present value. In addition, the discount rate at which the cash flows are discounted has fallen, increasing their present value still further. Note that, in common with MBS, and unlike a noncallable coupon bond, the value of a PO displays negative convexity at low interest rates. Whereas at high interest rates, when little or no prepayment is occurring, the PO behaves like a long-term (high duration) coupon bond, as interest rates fall, and mortgage holders start to prepay, the bond behaves more like a short-term bond, as most of the payments are likely to occur within a shorter period of time. This reduces the impact of a 1% drop in interest rates, leading to the observed negative convexity. IO securities, on the other hand, display a very pronounced *positive* relationship (*negative* duration) between price and interest rates. IO holders receive no principal payments at all, only interest. In the best possible state of the world, where mortgage holders never prepay their loans, IO holders receive 360 months of scheduled interest payments.

However, prepayment has the effect of cutting off payments on the date of prepayment, with no compensation in the form of an early return of principal (as for a MBS). Thus, in the extreme, an IO holder can go from expecting to receive 360 payments to receiving absolutely nothing in a short period of time. As interest rates fall, although the expected cash flows to IO holders are discounted at a lower rate, this is more than offset by the reduction in the number of these payments caused by increased prepayments. This variation is extreme. For example, the value of the 10.5% IO security fell by more than 50% over the period 1991–92, as interest rates fell from 9% to around 6%.

The other feature of these graphs is that, although there is a strong relationship between the prices of the assets and the level of the 10-year Treasury rate, this relationship is by no means perfect. There is a wide spread of prices for a given interest rate. For example, at an interest rate of 8%, we see prices for the *same* 10.5% IO security ranging between approximately $30 and $40 over the sample period. This variation clearly needs to be understood. Some candidate explanations include

Seasoning Does the age of the mortgage pool have a significant impact?

Second interest rate factor Do we need another interest rate factor (say a shorter rate) to describe the state of the world adequately?

Burnout Over time, as "fast prepaying" mortgage holders prepay their loans first, the composition of the pool changes.

3 Previous Research

Valuing stripped MBS is generally performed by taking a pricing model for a mortgage, or pass-through MBS, then splitting up the cash flows in the appropriate manner to calculate strip prices. The underlying MBS is roughly equivalent to a level payment, non-callable bond, minus a call option on that bond, with time-varying exercise price equal to the remaining principal on the loan at any instant. MBS pricing models have fallen into two classes. Rational models of mortgage holders' prepayment decisions, usually employing a simple, single factor model of interest rates, started with Dunn and McConnell [7, 8], who apply an option pricing approach to the valuation of MBSs. This approach also determines the optimal prepayment strategy as part of the MBS valuation process. The model, however, has some unattractive implications. First, the price of an MBS can never exceed par. Second, all mortgage holders refinance at the same time, as soon as interest rates fall below a critical level. To correct the first problem, Timmis [23], Dunn and Spatt [9], and Johnston and Van Drunen [13] add transaction costs which must be paid by borrowers should they decide to refinance their loans early. To relax the second restriction, Stanton [22] extends the model to include heterogeneous transaction costs across mortgage holders, and to permit mortgage holders to refinance only at discrete intervals, rather than continuously. There is thus no longer a particular interest rate level which induces uniform prepayment, and the model produces prepayment behavior that can exhibit most of the main features of real prepayment behavior, including *seasoning*[9] and a mortgage pool's *burnout*[10] (see Richard and Roll [19] for a discussion of various other factors affecting prepayment rates).

As an alternative, Schwartz and Torous [20] (and many Wall Street firms) use a Monte Carlo approach to price MBSs. An empirical prepayment model is used, which predicts prepayment in any given month as a function of some set of explanatory variables (usually including interest rates and lagged prepayment). Various interest rate paths are simulated, and the prepayment model is used to predict the cash flow each month along each interest rate path. The predicted cash flows are then discounted at the appropriate risk-adjusted rate, and the average across all simulated paths is used as an estimate of the security price. While this approach allows greater flexibility in the form of the prepayment function than the rational models, this comes at a cost. With no economic guidance as to the correct set of variables to use, there is always the danger of finding spurious relationships between variables that are in fact independent. With no restrictions on the functional form to be used, there

[9] Prepayment rates on mortgages initially tend to increase with the age of the mortgage, since there are frictions to household changes. For example, brand new mortgages are unlikely to have been taken out if the holders thought they were to get divorced, relocate or default.

[10] That is, for aged (and substantially prepaid) pools in a positive coupon spread environment, there is a tendency for low future prepayments. The intuition is that if a mortgage holder were going to prepay, then he/she would have already done so. This *burnout* effect could reflect nonoptimal behavior on the part of some mortgage holders, or frictions they face in trying to refinance their property (e.g. the value of the house may have fallen by such an amount that refinancing is no longer possible, yet there are sufficient costs to defaulting).

is also a great danger of misspecification. Both of these can have serious consequences for asset pricing and hedging.[11]

Previous attempts to price SMBS have followed a similar pattern. O'Brien [18] derives simple closed form expressions for IO and PO values assuming a constant prepayment rate. While this obviously fails to take into account the relationship between prepayment rates and interest rate movements, it shows how sensitive IO and PO prices are to changes in assumptions about prepayment rates. Schwartz and Torous [21] combine their empirical prepayment model (Schwartz and Torous [20]) with the Brennan and Schwartz [4] two factor interest rate model, using Monte Carlo simulation to calculate values for stripped mortgage-backed securities. Rather than looking at pure IO/PO strips, they consider forming two synthetic securities from an 11% mortgage-backed security. Each new security receives 1/2 of the principal payments from the 11% security, but the interest payments are split in the ratio 7:4, resulting in synthetic 14% and 8% securities, whose value depends on the prepayment of the underlying 11% security. They find that, given the assumptions of their model, the value of the synthetic securities can differ markedly from the value of a standard 14% or 8% mortgage-backed security. McConnell and Singh [15] use a similar Monte Carlo approach to price more complex CMOs, also combining the Schwartz and Torous [20] prepayment model with the Brennan and Schwartz [4] two factor interest rate model. As examples, they consider CMOs with IO and PO tranches (though these are not the same as IO and PO strips, since these are still part of the sequentially paying CMO structure). More recently, McConnell and Singh [16] combine the rational prepayment model of Stanton [22] with their simulation algorithm, to examine the impact of changing their assumptions about the interest rate process[12] on CMO prices. Yang [25] also assumes the Cox, Ingersoll and Ross [5] interest rate model, and values IO/PO strips assuming mortgage holders refinance optimally, subject to a transaction cost to be paid on refinancing. He also allows two types of "suboptimal" prepayment. With some probability, when optimal to refinance, mortgage holders choose not to, and with some other probability, they refinance when it is not optimal to do so.

[11]For example, Breeden [3] shows that even the most sophisticated Wall Street forecasts of prepayment rates and interest rate sensitivity of MBS and SMBS were often dramatically wrong over the period 1992–94.

[12]They use an extended Cox, Ingersoll and Ross [5] model, due to Hull and White [12]; an extended Vasicek [24] model; and the Heath, Jarrow and Morton [11] model.

An alternative approach to the valuation and hedging of mortgages and mortgage-backed securities has recently been developed by Boudoukh, Richardson, Stanton and Whitelaw [2, 1]. Rather than explicitly modeling either the process governing interest rate movements or the prepayment behavior of mortgage holders, they directly estimate (nonparametrically) the relationship between security prices and interest rates. This avoids the possibility of misspecification, though at the cost of potentially greater estimation error. While this approach is useful in the context of pass-through mortgage-backed securities, especially newly issued securities, where an assumption of stationarity in the relationship between prices and interest rates is reasonable, it is harder to justify in the present context. Just as the values of IO and PO securities are more sensitive to differing prepayment assumptions than are the underlying mortgages, so will their values depend more on factors such as time since issue (due both to its impact on prepayment behavior and on the changing split between interest and principal caused by the loan's amortization schedule), and burnout (due to its impact on prepayment). While, in principle, additional state variables could be added to capture time since issue and some measure of burnout, the usefulness of nonparametric techniques lessens as the dimensionality of the problem increases, and the need to choose a specific burnout variable means that misspecification would again be an issue, even in a nonparametric context.

These models give us valuable intuition into the factors governing the valuation of IO/PO strips and other mortgage derivatives. However, none of these authors performs any empirical comparison of (parametric) model and market prices. Their results show, however, that the prices produced by these models are extremely sensitive to the assumptions made, emphasizing the importance of correct model specification and estimation. The next section describes the main features of the Stanton [22] prepayment and valuation model.

4 The Model

4.1 Modeling Prepayment

Assume mortgage holders minimize the market value of their mortgage liabilities by optimally exercising their prepayment options (subject to certain frictions). Defining F_t to be the remaining principal on the loan at time t, assume mortgage holder i must pay a proportional transaction cost $X_i F_t$ on refinancing. This includes both direct monetary costs,[13] and the value of non-monetary components reflecting the difficulty and inconvenience of filling out forms, lost productivity, etc. Let B_t be the value of a non-callable bond which makes payments equal to the promised payments on the mortgage. The mortgage holder has a call option on B_t with time varying exercise price $F_t(1+X_i)$. The value of the mortgage liability, M_t^ℓ, is

$$M_t^\ell = B_t - V_t^\ell, \tag{1}$$

where V_t^ℓ is the value of the prepayment option to the mortgage holder. Since B_t does not depend on the mortgage holder's prepayment decision, minimizing the liability value is equivalent to maximizing the option value.

Besides refinancing for interest rate reasons, the mortgage holder may also prepay for exogenous reasons, such as divorce, job relocation, or sale of the house. The likelihood of exogenous prepayment is described by a hazard function λ. Informally, the probability of prepayment in a time interval of length δt, conditional on not having prepaid prior to t, is approximately $\lambda\,\delta t$. The parameter λ represents a baseline prepayment level, the expected prepayment level when no interest rate driven refinancing should occur.

Assume that mortgage holders decide whether to prepay their mortgage *not* continuously, but rather at random discrete intervals. This would result, for example, from mortgage holders facing some fixed cost payable when making each decision. Assume the likelihood of making a prepayment decision is governed by a hazard function ρ.[14]

The value of a mortgage-backed security, whose cash flows are determined by the prepayment behavior of the mortgage holder, is $M_t^a = B_t - V_t^a$ (a for "asset"). As noted by Dunn and Spatt [9], there is a difference between the asset and liability values because of the transaction costs associated with prepayment. While these are paid by the mortgagor, and thus increase the value of the liability (reducing the value of the option), they are not received by the investor in the mortgage-backed security. The two values must be calculated simultaneously, since the optimal prepayment strategy of the mortgage holder, determined as part of

[13] Appraisal, title insurance, credit check, etc.

[14] The models of Dunn and McConnell [8], Timmis [23], Dunn and Spatt [9], and Johnston and Van Drunen [13] implicitly set $\rho = \infty$.

the liability valuation, in turn determines the cash flows that accrue to the mortgage-backed security.

Each period, given the current interest rate and the transaction cost level of the mortgage holder, the optimal prepayment strategy[15] determines whether the mortgage holder should refinance. For a given coupon rate, and transaction cost X_i, there is a critical interest rate r_t^* such that if $r_t \leq r_t^*$ the mortgage holder will optimally choose to prepay. Equivalently, for a given coupon rate and interest rate r_t, there is a critical transaction cost X_t^* such that if $X_i \leq X_t^*$ the mortgage holder will optimally prepay. If it is not optimal for the mortgage holder to refinance, any prepayment is for exogenous reasons, so the hazard rate governing prepayment equals λ. If it *is* optimal to refinance, the mortgage holder may prepay in the next time interval either for interest rate related or for exogenous reasons. The probability that the mortgage holder does *not* prepay in a (small) time interval of length δt is the probability of neither prepaying for exogenous reasons, nor making an interest rate related prepayment decision during this period,

$$e^{-\lambda\,\delta t} e^{-\rho\,\delta t} = e^{-(\lambda+\rho)\,\delta t}. \tag{2}$$

As δt goes to zero, the probability of prepayment approaches $(\lambda+\rho)\,\delta t$. Thus the hazard rate governing prepayment at time t, with interest rate r_t equals

$$\begin{cases} \lambda & \text{if} \quad X_i > X_t^*, \\ \lambda+\rho & \text{if} \quad X_i \leq X_t^*. \end{cases} \tag{3}$$

Finally, assume that, in a given pool of mortgage holders, the transaction costs X_i faced by the individual mortgage holders are initially distributed according to a beta distribution with parameters α and β. The mean and variance of the beta distribution are

$$\begin{aligned} \mu &= \frac{\alpha}{\alpha+\beta}, \\ \sigma^2 &= \frac{\alpha\beta}{(\alpha+\beta)^2\,(\alpha+\beta+1)}. \end{aligned}$$

[15]This is determined as part of the mortgage valuation process - see Stanton [22] for details.

Given the distribution of transaction costs in a pool,[16] it is now a simple matter to value a mortgage backed by the pool, and to determine the expected prepayment rate each month, given values for the parameters λ, ρ, α and β. Since the cash flow from the pool is the sum of the cash flows from the individual mortgages, we just need to value each type of mortgage in the pool, and weight each value by the fraction of the pool of that type, keeping track of how the composition of the pool changes over time.[17]

4.2 Interest Rates

To solve the model we must make assumptions about the process governing interest rate movements. We use the Cox, Ingersoll and Ross [5] one-factor model to characterize nominal interest rate movements. In this model, the instantaneous risk-free interest rate r_t satisfies the stochastic differential equation

$$dr_t = \kappa(\mu - r_t)\,dt + \sigma\sqrt{r_t}\,dz_t. \tag{4}$$

This equation says that, on average, the interest rate r converges toward the value μ. The parameter κ governs the rate of this convergence. The volatility of interest rates is $\sigma\sqrt{r_t}$. One further parameter, q, which summarizes risk preferences of the representative individual, is needed to price interest rate dependent assets.

The four parameters of the interest rate process were estimated using daily 3-month and 10-year Treasury rates over the period June 1990–January 1995.[18] The estimated parameter values over this period are

$$\begin{aligned}
\kappa &= 0.3089, \\
\mu &= 0.03827, \\
\sigma &= 0.03474, \\
q &= -0.2190.
\end{aligned}$$

[16] For implementation purposes, a discrete version of that distribution is used

[17] As interest rates fall, those mortgage holders with lower refinancing costs will find it optimal to refinance first. As a result, the average transaction cost faced by mortgage holders in the pool will increase over time.

[18] This matches the period of the IO/PO prices.

Given this model for movements in r_t, the value of an an interest rate contingent claim paying coupons or dividends at some rate $C(r_t, t)$, satisfies the partial differential equation[19]

$$\frac{1}{2}\sigma^2 r V_{rr} + [\kappa\mu - (\kappa + q)r]\, V_r + V_t - rV + C = 0. \tag{5}$$

Solving this equation, subject to appropriate boundary conditions, gives the asset value $V(r_t, t)$, and (in the case of assets with embedded options) the optimal exercise strategy. We solve equation (5) for mortgage, IO and PO prices using the Crank-Nicholson finite difference algorithm.[20]

5 Estimation of the Model

To estimate the model's parameters λ, ρ, α and β, we find the set of parameters which makes the IO and PO prices produced by the model as close as possible to market prices. Specifically, we minimize the sum of squared pricing errors over the period 1990–95, and across both IO and PO securities. In other words, defining $\boldsymbol{\theta} = (\rho, \lambda, \alpha, \beta)$, our estimator, $\widehat{\boldsymbol{\theta}}$, is the value which minimizes the sum of squared residuals,

$$S_T(\boldsymbol{\theta}) = \sum_{c=c_1}^{c_n} \sum_{t=1}^{T} \left[(\mathrm{IO}_t^c - \widehat{\mathrm{IO}}_t^c(\widehat{\boldsymbol{\theta}}))^2 + (\mathrm{PO}_t^c - \widehat{\mathrm{PO}}_t^c(\widehat{\boldsymbol{\theta}}))^2 \right], \tag{6}$$

where IO_t^c is the market price at time t of the benchmark IO security with coupon rate c, $\widehat{\mathrm{IO}}_t^c(\widehat{\boldsymbol{\theta}})$ is the corresponding model price, and similarly for the PO prices. Assuming homoscedastic pricing errors, with constant variance, the asymptotic variance-covariance matrix for the estimated parameter vector is

$$\frac{S_T(\widehat{\boldsymbol{\theta}})}{2nT} \left[\sum_{c=c_1}^{c_n} \sum_{t=1}^{T} \left(\frac{\partial \widehat{\mathrm{IO}}_t^c(\widehat{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta}} \frac{\partial \widehat{\mathrm{IO}}_t^c(\widehat{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta}'} \right) + \left(\frac{\partial \widehat{\mathrm{PO}}_t^c(\widehat{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta}} \frac{\partial \widehat{\mathrm{PO}}_t^c(\widehat{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta}'} \right) \right]^{-1}. \tag{7}$$

[19] We need to assume some technical smoothness and integrability conditions [see, for example, Duffie [6]].

[20] See McCracken and Dorn [17] for a description of the algorithm, and Stanton [22] for details of its application to pricing mortgages and determining the optimal exercise strategy.

5.1 Estimation Results

We focus first on the 10.5% coupon securities.[21] Using the estimation procedure described above, we estimated the parameters of the prepayment model, obtaining the results

$$\begin{aligned}\rho &= 0.461, \\ \lambda &= 0.0016, \\ \alpha &= 1.221, \\ \beta &= 19.66.\end{aligned}$$

The estimated value of ρ tells us that when it is optimal for a mortgage holder to refinance, it takes an average of $1/\rho = 2$ years and 2 months before refinancing actually occurs. The low value for λ says that, on average, 0.16% of the mortgage holders in the pool prepay each year for exogenous (non-interest related) reasons. The values of α and β, which describe the initial distribution of refinancing costs faced by mortgage holders in the pool, imply an average refinancing cost of $\alpha/(\alpha+\beta) = 5.8\%$. This is substantially lower than the results obtained in Stanton [22], indicating perhaps that the publicity surrounding the refinancing booms of the last few years, combined with simplification of the refinancing process itself (more available types of loan to choose from, less documentation required, etc.) has lowered the effective barriers to refinancing faced by mortgage borrowers.

Figure (4) shows fitted vs. actual prices for both 10.5% IO and PO securities over the sample period, together with the 10-year Treasury yield. This graph shows several things. First, the model seems to match prices better in the second half of the sample period than in the first. In 1990–91 (when interest rates were at their highest) both IO and PO prices display significant variation from the prices predicted by the model. These differences become less pronounced over time, with model and market prices much closer together in the latter half of 1994. Figure (5) shows a scatter plot of both fitted and actual IO and PO prices, plotted against the 10-year yield. The main thing to observe here is that the plots for the

[21] This coupon is chosen because we have data for the entire sample period, the benchmark trust does not change over the period, and the IO and PO prices display significant variation when plotted against the 10-year rate.

fitted prices, both IO and PO, are very narrow. Despite the ability of the model to exhibit burnout, it is clear that, at the estimated parameter values, the model generates IO and PO prices which are almost exact functions of the 10-year rate. The actual prices, on the other hand, display much wider variation. Figure (6) looks at the implied MBS prices, obtained by adding the prices (both fitted and actual) of the individual IO and PO securities. To aid comparison, the scale is the same as for the graphs of IO and PO prices separately. It can immediately be seen that the general interest rate sensitivity of the (implied) MBS is much lower than that of either the IO or PO securities separately. The difference between the actual and fitted prices is less volatile than with the IO and PO prices, but the model prices have a consistent positive bias. This is seen dramatically in figure (7), which plots IO pricing errors against PO pricing errors. There is a very strong negative correlation between the two. While the model is doing a reasonable job of capturing the behavior of the MBS, the main source of the pricing errors seems to be in its splitting that value between IO and PO pieces.

Figures (8)–(16) investigate the pricing errors further, looking in particular at some of the possible factors that might be expected to cause them. Figures (8) and (9) plot pricing errors and the 10-year rate over time for IO and PO securities respectively. There are some interesting patterns in the errors. The IO errors tend to move *with* interest rates in the early part of the sample (1990 to mid-1992), then *against* interest rates from mid-1992 to mid-1994, then *with* interest rates again from mid-1994 on. These periods coincide with interest rates being greater than 7% (errors move with interest rates) and less than 7% (errors move against interest rates). The story is reversed for the PO errors. In the early and late parts of the sample (interest rates above 7%), PO errors move against the direction of interest rates. In the middle period (interest rates below 7%), PO errors move with interest rates. Figure (10) shows the corresponding behavior of the aggregate errors. These display similar behavior to the IO errors, but are much less volatile. For example, the large swing in both IO and PO errors at the end of 1990 is almost absent from the aggregate results.

To analyze the source of the errors further, figures (11)–(16) are scatter plots of the pricing errors against possible explanatory factors. Figures (11) and (12) plot IO and PO errors against the 10-year rate. There does not seem to be a strong relationship in either case. The next four figures look at whether a second interest rate factor is the source of the errors. Figures (13) and (14) plot the errors against the difference between the 3-month Treasury rate and the 3-month rate implied by the Cox, Ingersoll and Ross interest rate model, given the level of the 10-year rate.[22] Figures (15) and (16) plot the errors against the slope of the yield curve.[23] None of these graphs shows a strong relationship.

Finally, figure (17) shows predicted vs. actual prepayment rates for the underlying mortgage pool over time. In the middle of the sample (1992 to mid-1994), while there is substantial volatility in the actual prepayment rates observed, they seem to be centered around the predicted level. However, before and after this period, the model predicts prepayment which is substantially higher than that seen in the data.

5.2 Nonparametric Analysis of Pricing Errors

To be written

5.3 Effect of Changing the Benchmark

To be written

6 Hedging

To be written

7 Summary

This paper investigates the pricing of interest only (IO) and principal only (PO) stripped mortgage-backed securities using a sample of market prices for these securities over the period June 1990–January 1995. We fit the parameters of the Stanton [22] mortgage prepayment

[22]This is a measure of how much our chosen interest rate model fails to capture the slope of the yield curve.

[23]The difference between the 10-year and 3-month rates.

and valuation model, and analyze the model's pricing and prepayment errors. Our results indicate several directions in which the valuation model needs to be extended.

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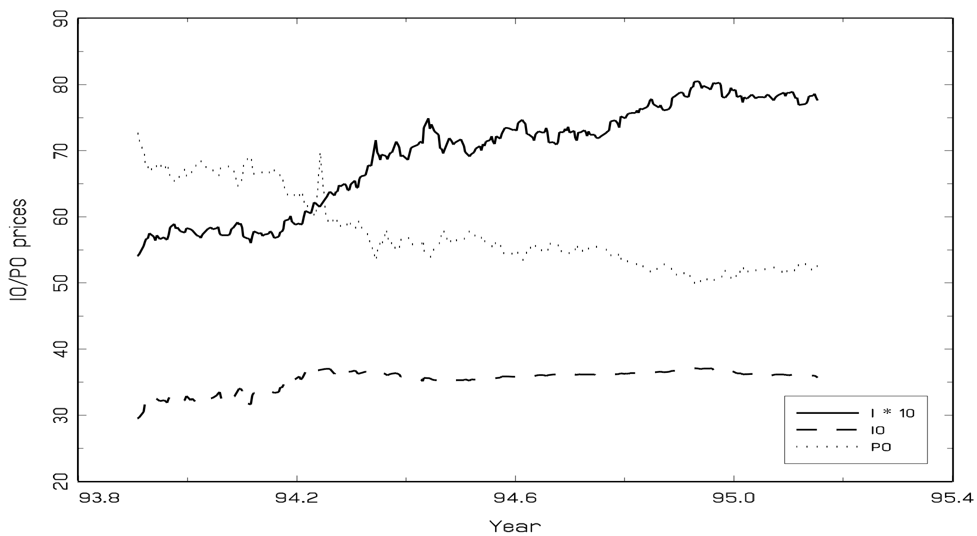
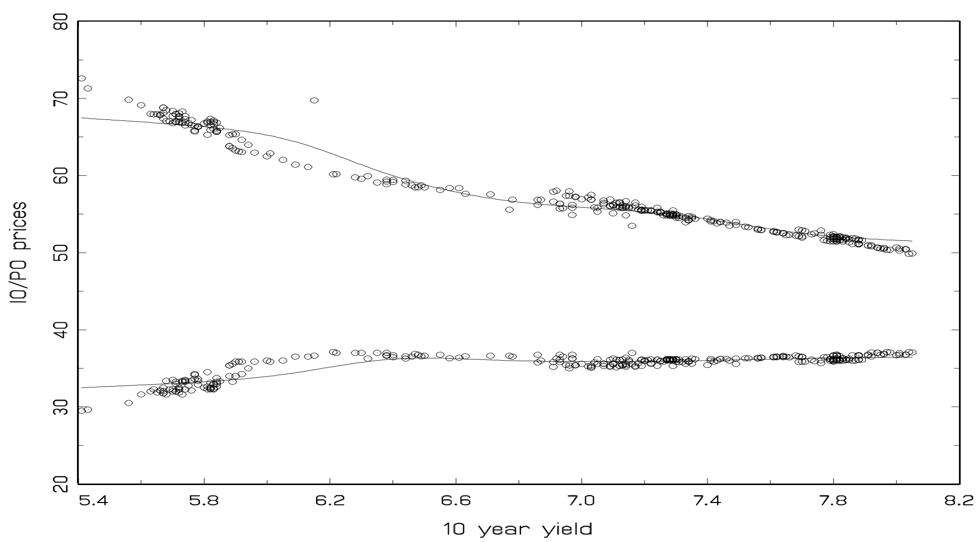
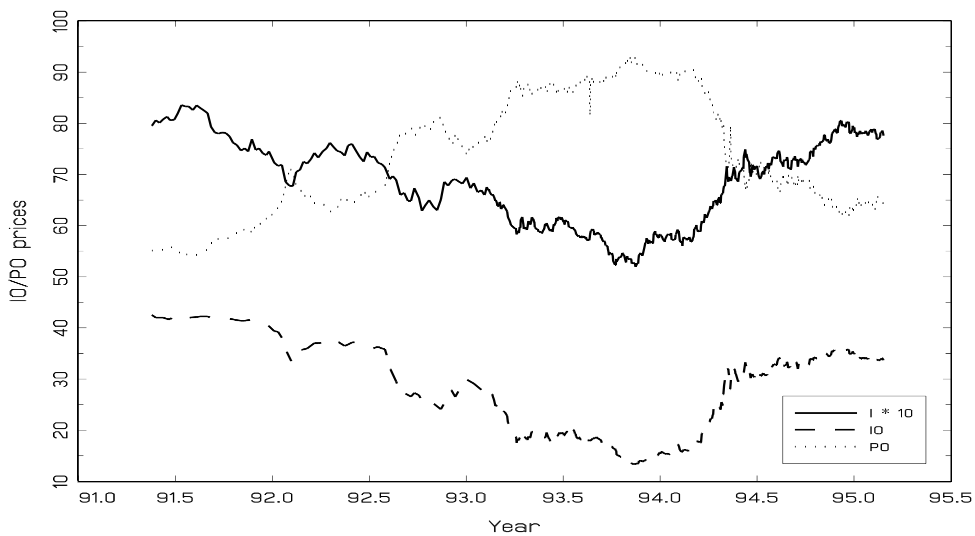
Figure 1: 6.5% IO and PO prices

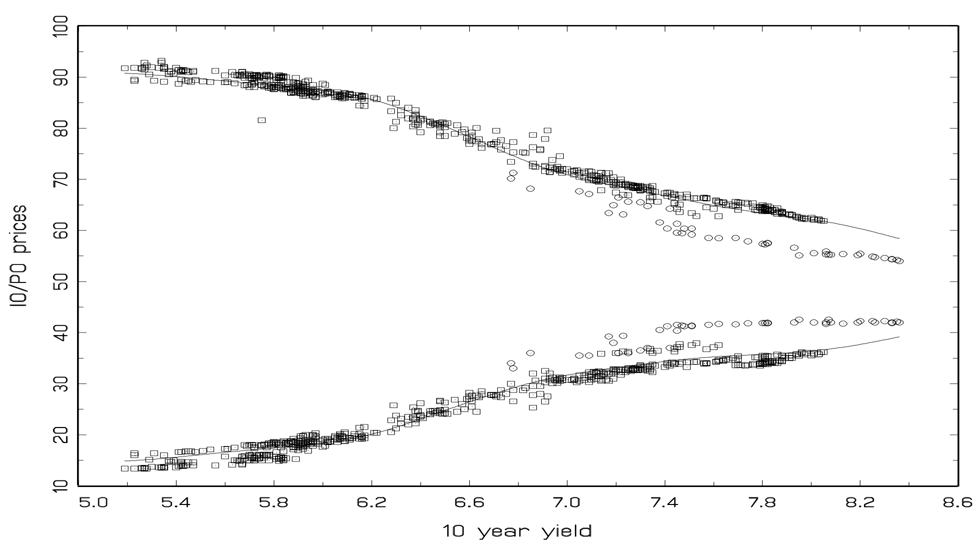




Prices of 6.5% IO and PO stripped mortgage-backed securities

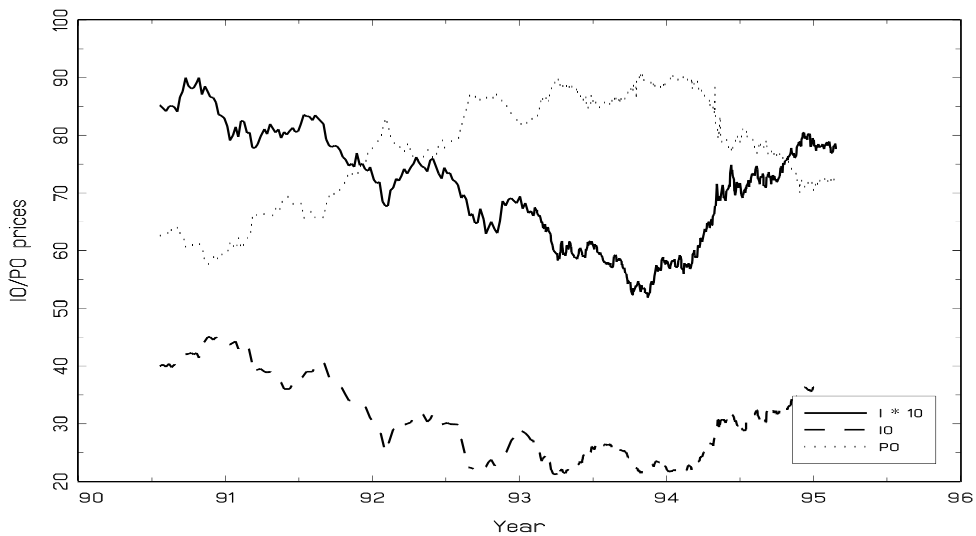
Figure 2: 8.5% IO and PO prices

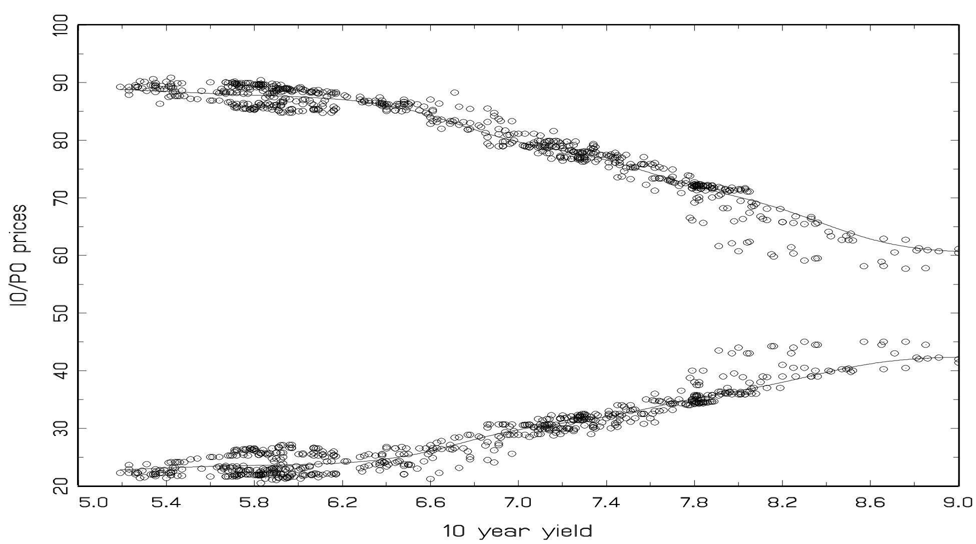




Prices of 8.5% IO and PO stripped mortgage-backed securities

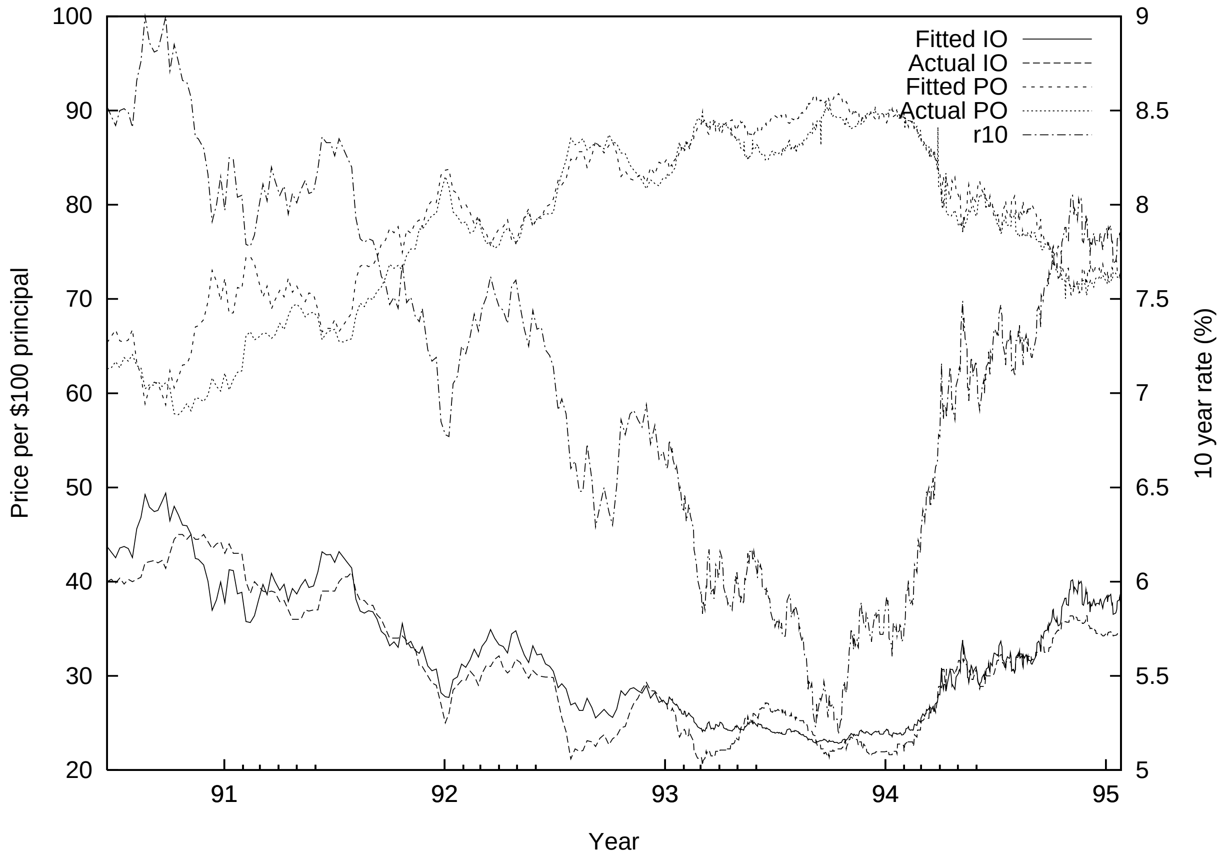
Figure 3: 10.5% IO and PO prices





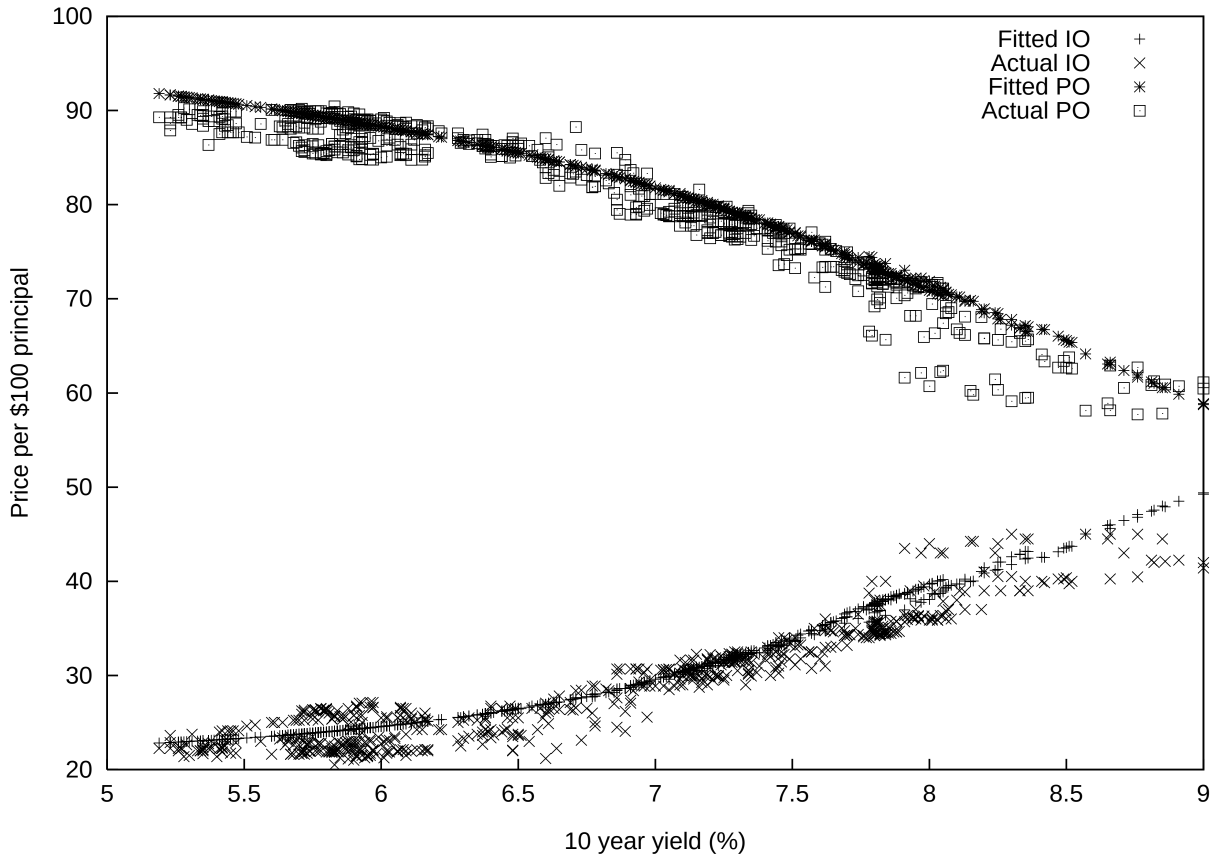
Prices of 10.5% IO and PO stripped mortgage-backed securities

Figure 4: Fitted vs. actual 10.5% IO and PO prices



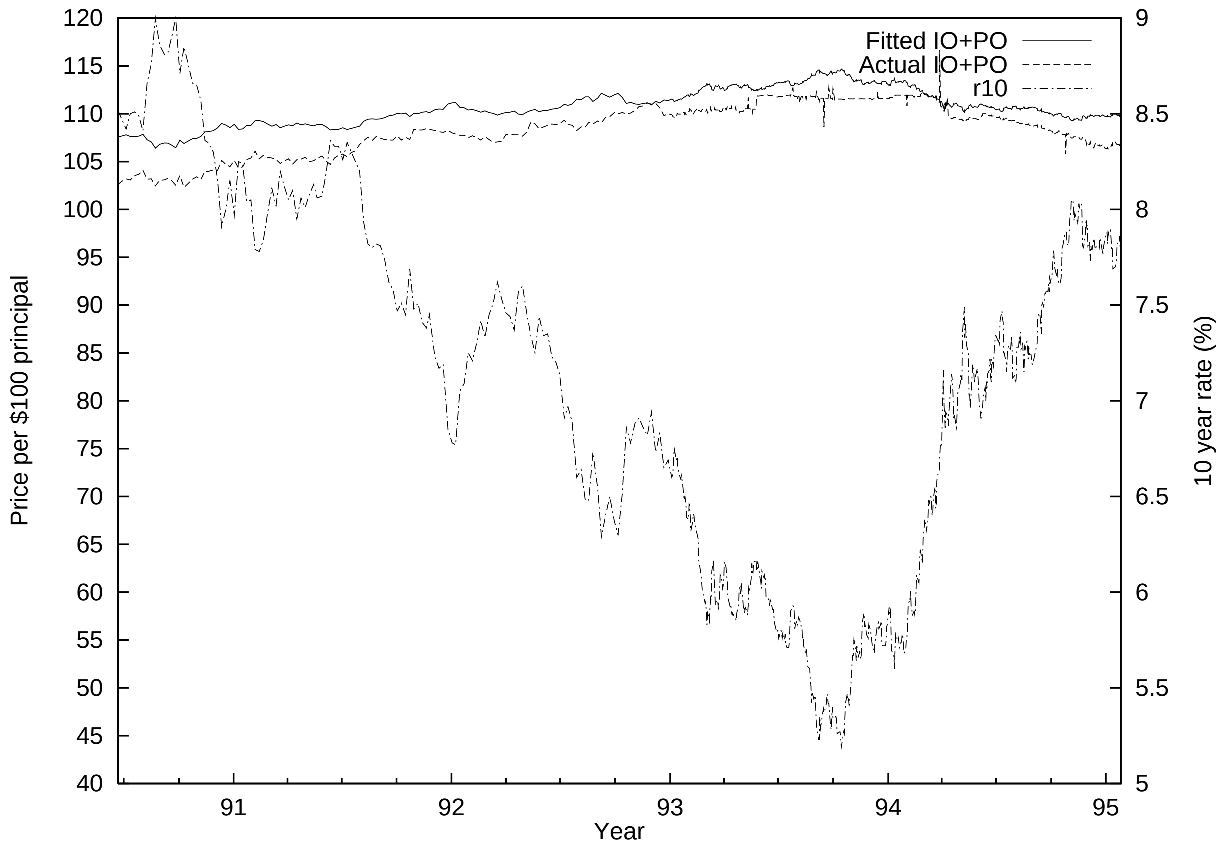
Fitted vs. actual 10.5% IO and PO prices

Figure 5: Fitted and actual 10.5% IO and PO prices vs. 10-year yield



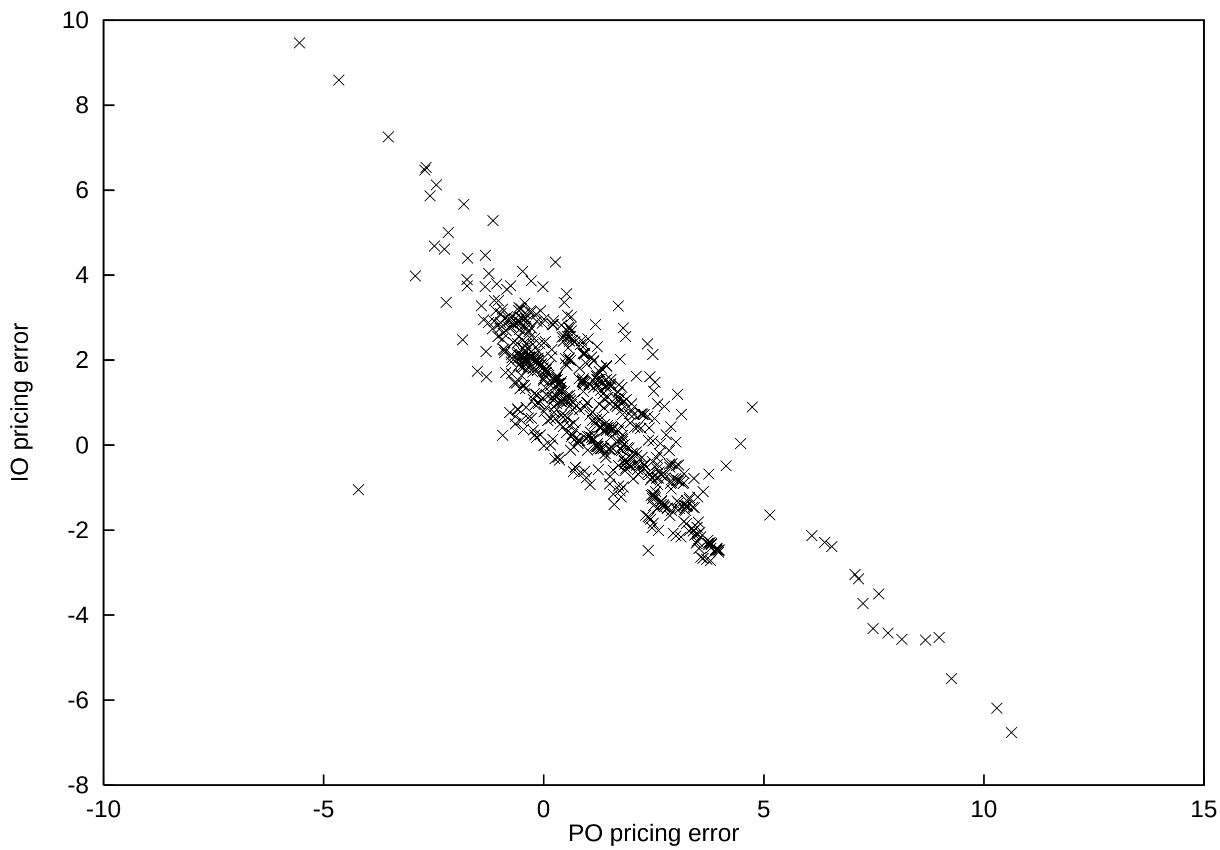
Fitted and actual 10.5% IO and PO prices vs. 10-year yield

Figure 6: Fitted vs. actual 10.5% IO+PO prices



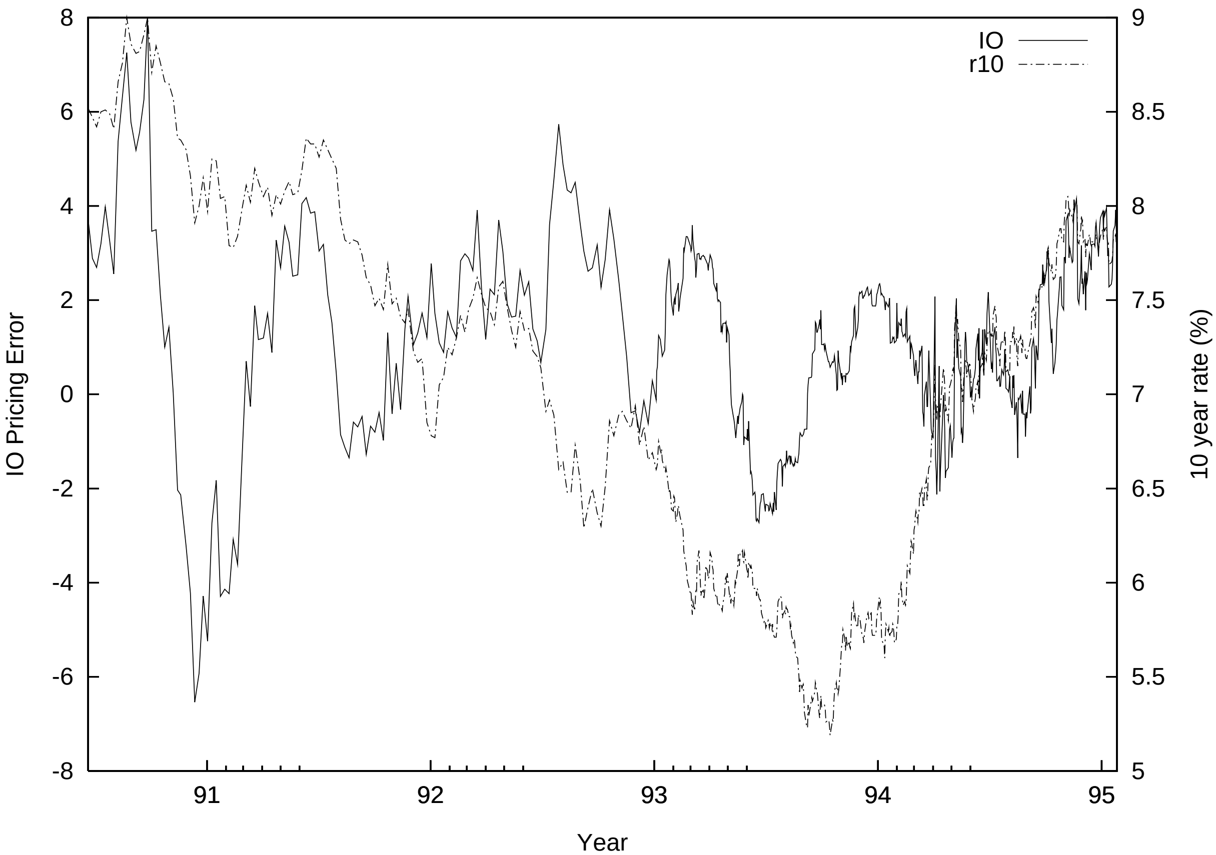
Fitted vs. actual 10.5% IO+PO prices

Figure 7: IO pricing errors vs. PO pricing errors



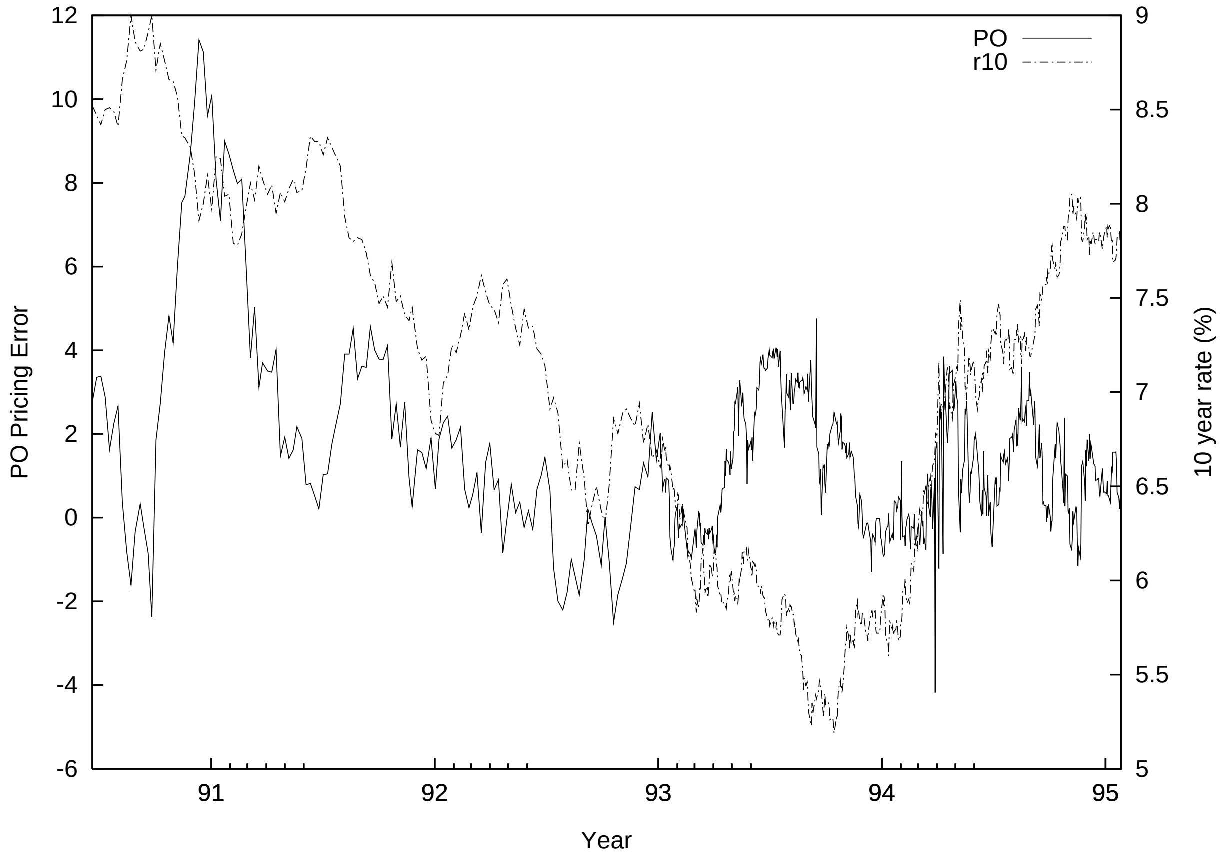
IO pricing errors vs. PO pricing errors

Figure 8: IO pricing errors



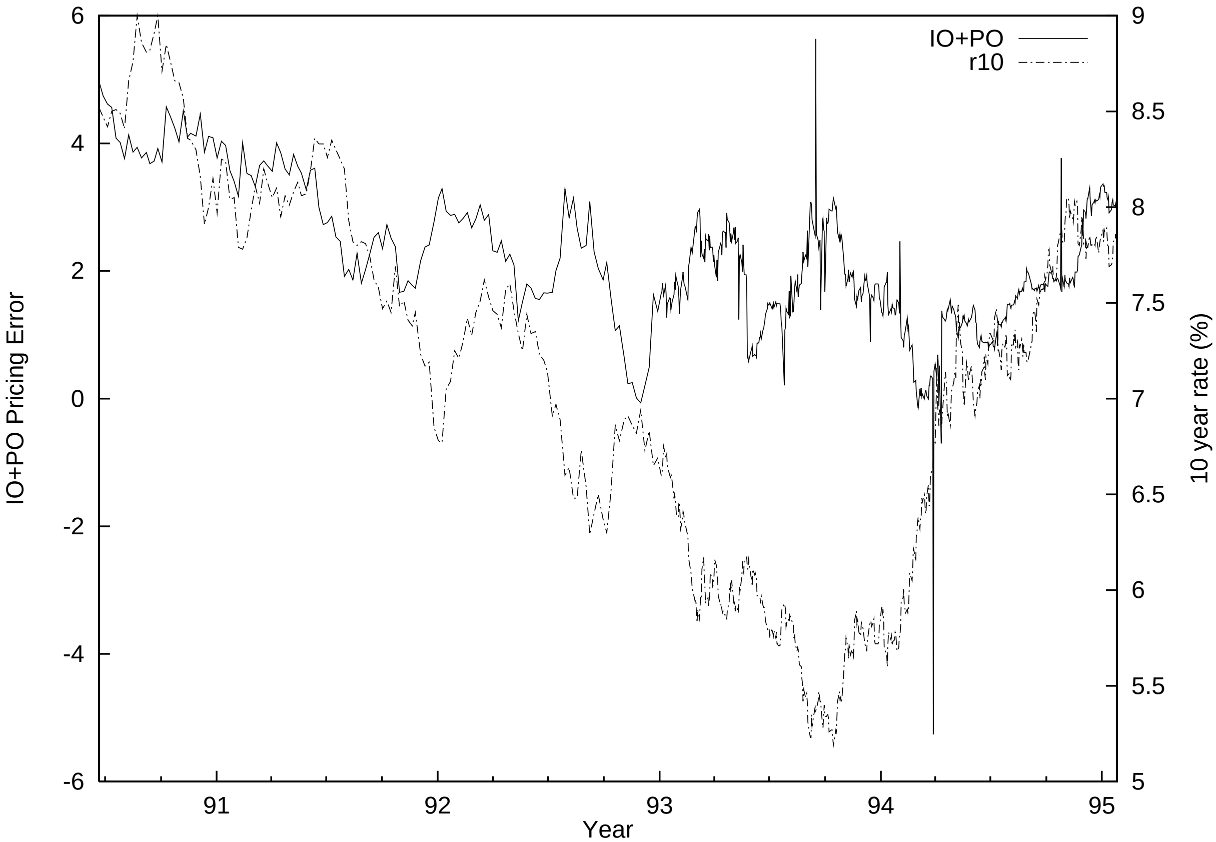
IO pricing errors

Figure 9: PO pricing errors



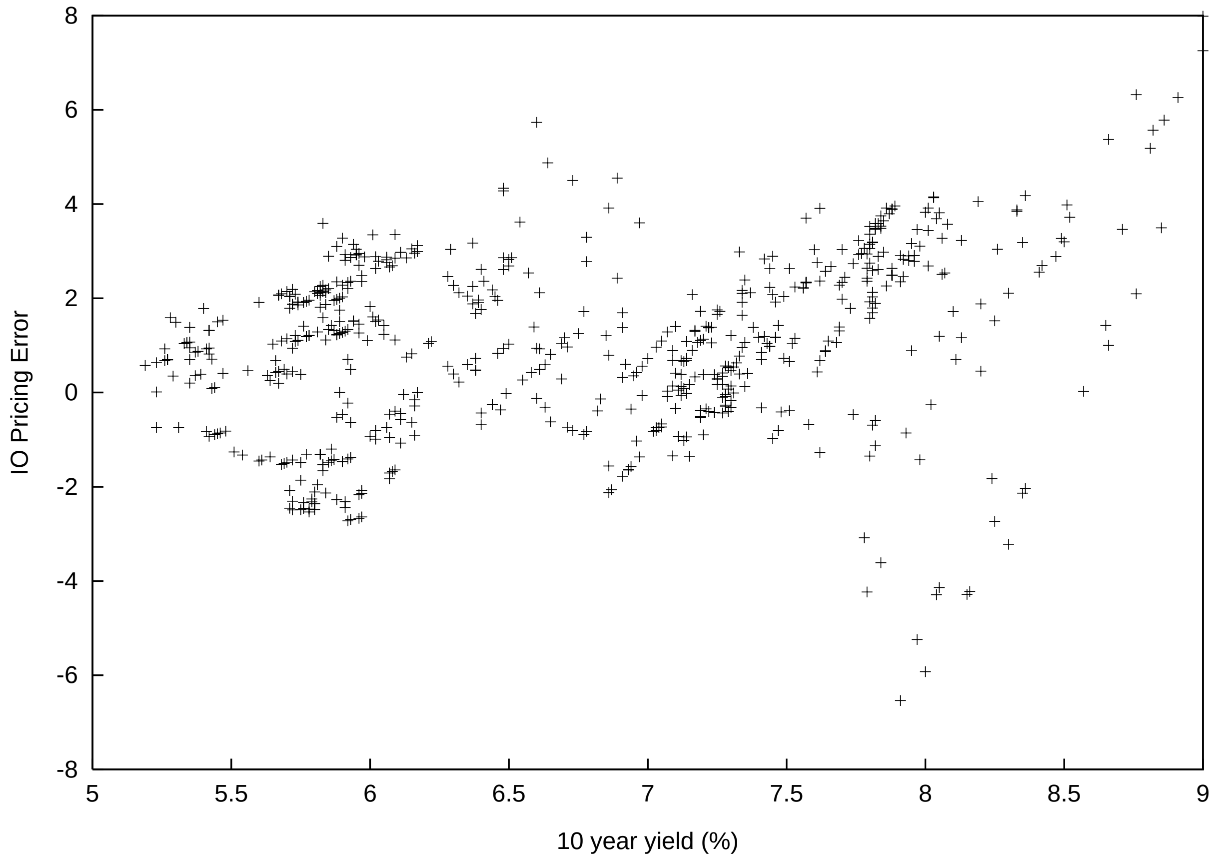
PO pricing errors

Figure 10: IO+PO pricing errors



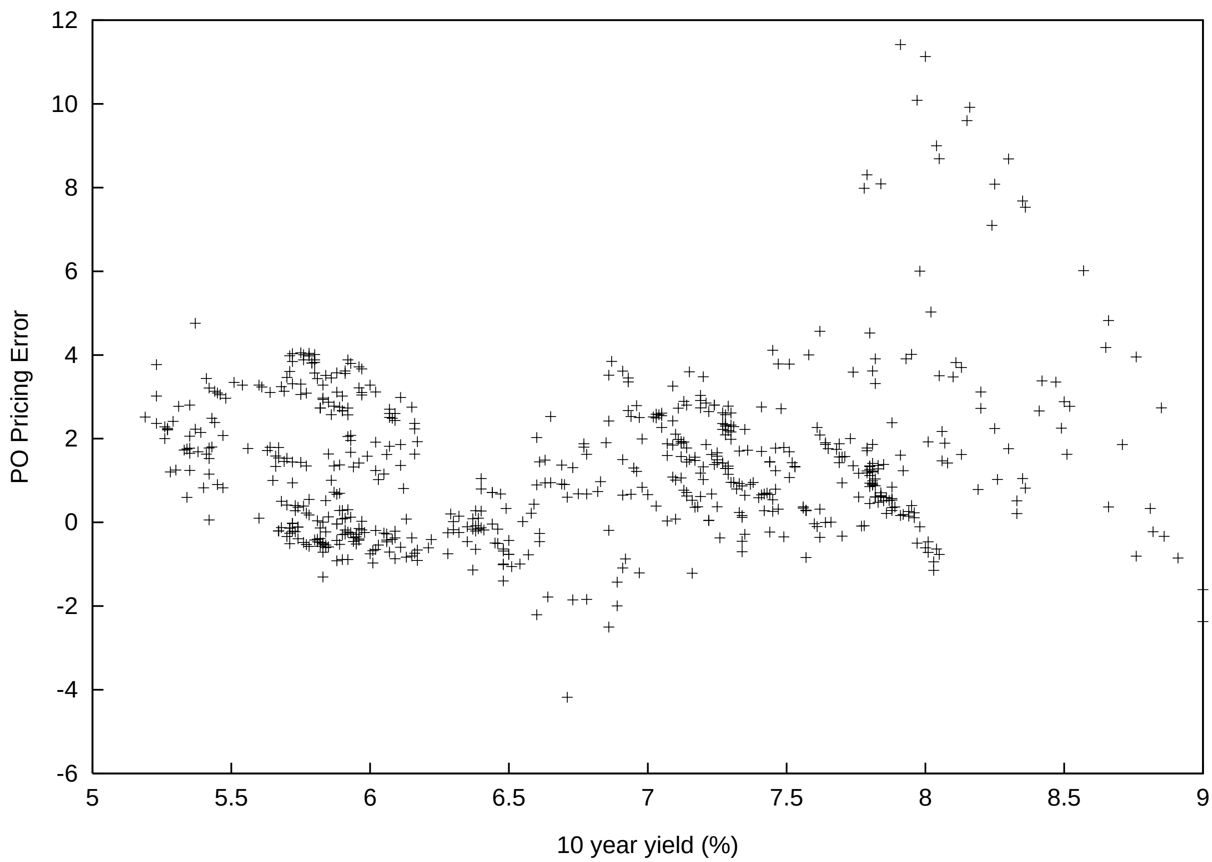
IO+PO pricing errors

Figure 11: IO pricing errors vs. 10-year rate



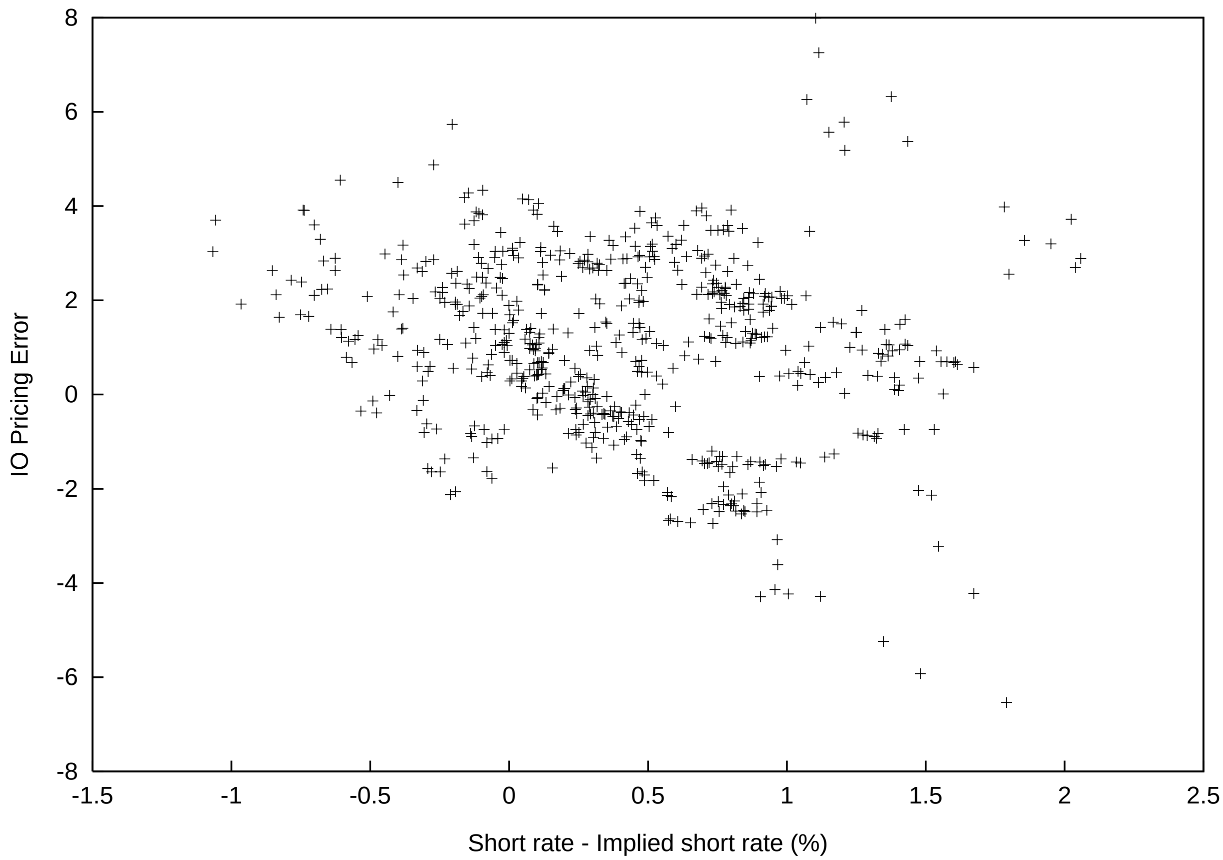
IO pricing errors vs. 10-year rate

Figure 12: PO pricing errors vs. 10-year rate



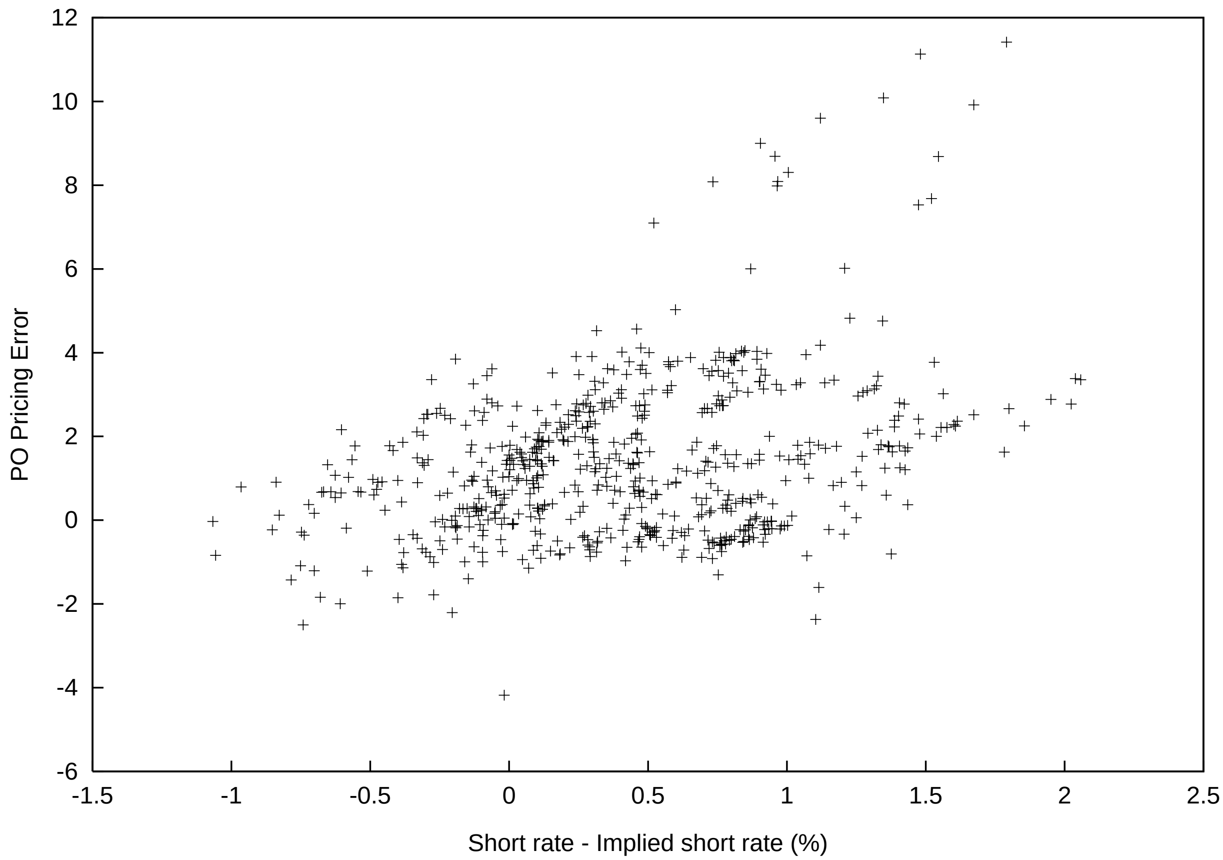
PO pricing errors vs. 10-year rate

Figure 13: IO pricing errors vs. actual - implied short rate



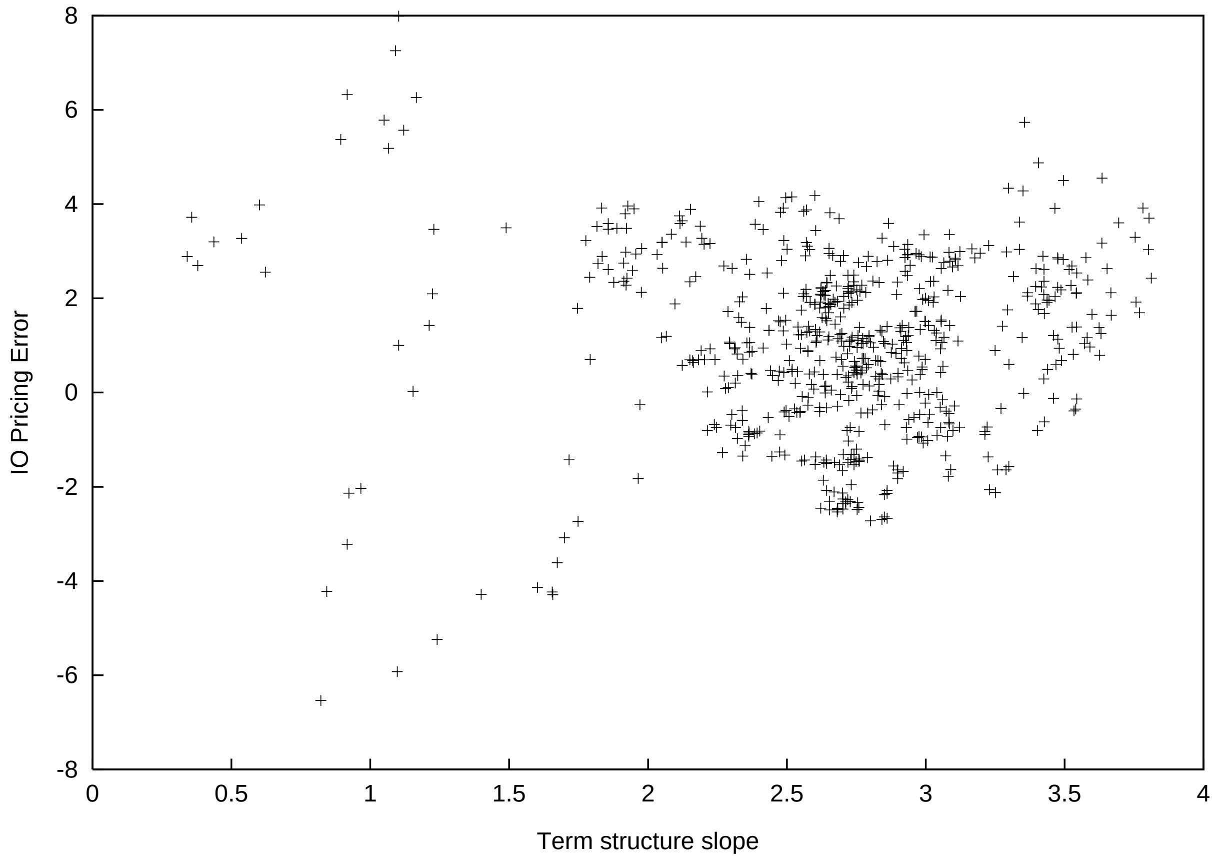
IO pricing errors vs. actual - implied short rate

Figure 14: PO pricing errors vs. actual - implied short rate



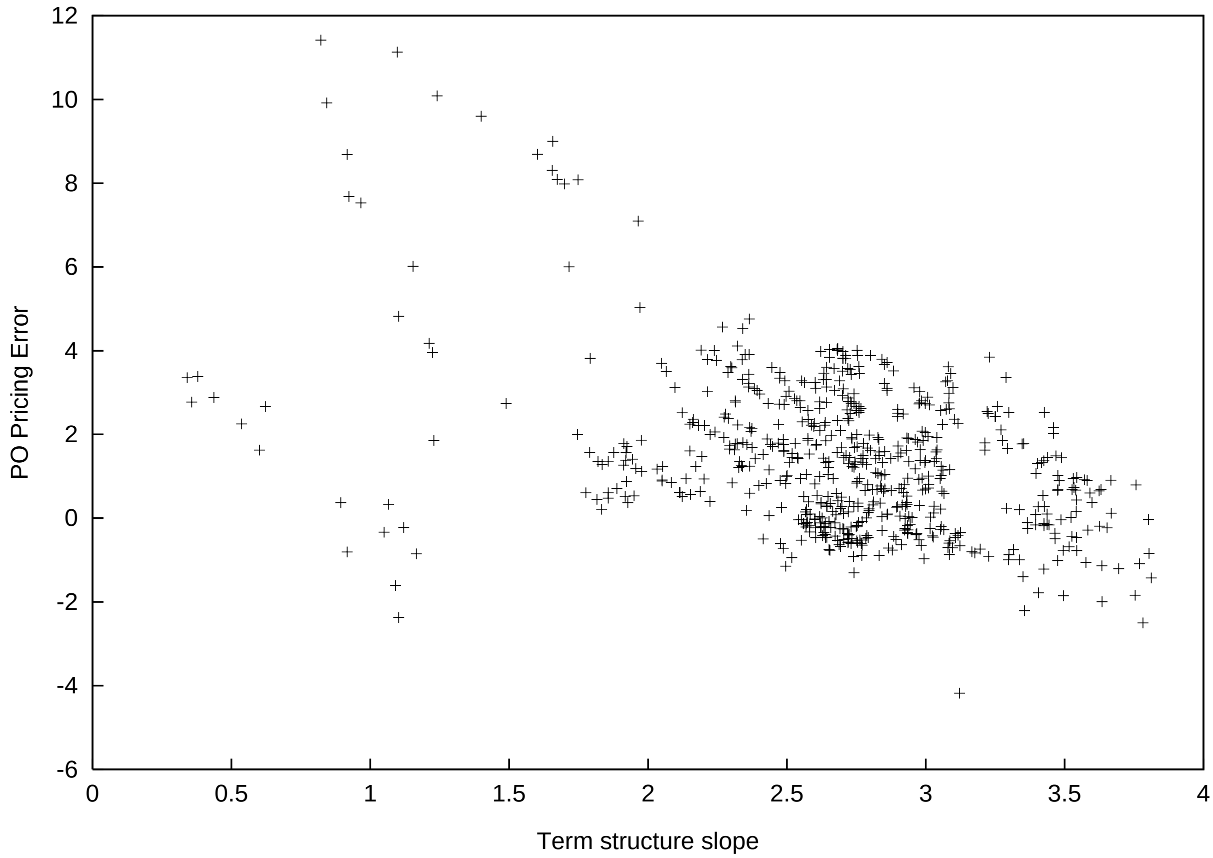
PO pricing errors vs. actual - implied short rate

Figure 15: IO pricing errors vs. term structure slope



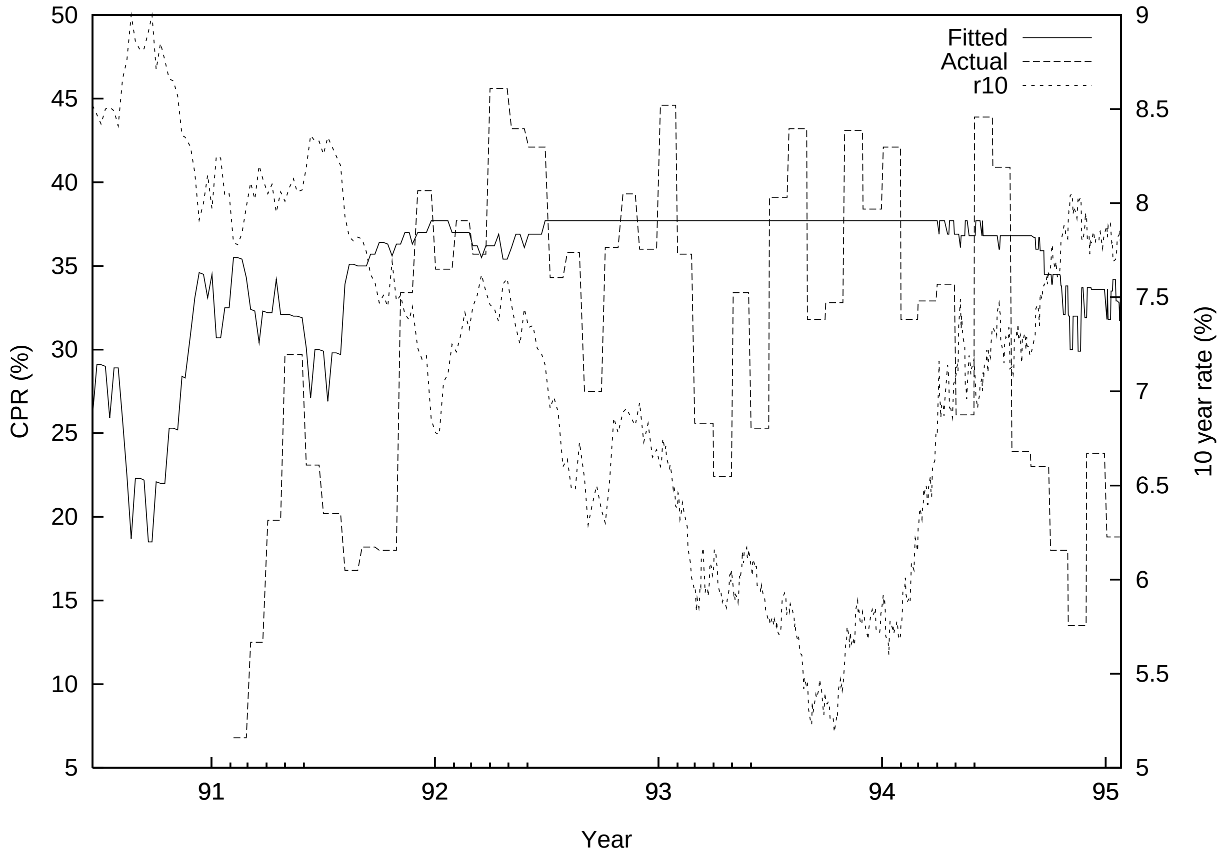
IO pricing errors vs. term structure slope

Figure 16: PO pricing errors vs. term structure slope



PO pricing errors vs. term structure slope

Figure 17: Actual vs. implied prepayment rates (CPR)



Actual vs. implied prepayment rates (CPR)

8 Notes/Other Papers

Marcus and Kling [14].