Journal of Financial Economics 34 (1993) 387-408. North-Holland

Is the *ex ante* risk premium always positive?

A new approach to testing conditional asset pricing models*

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Received November 1992, final version received May 1993

This paper develops tests of inequality restrictions implied by conditional asset pricing models. The methodology is easy to implement, requires little knowledge of the conditional distribution of asset returns, and is valid under fairly weak assumptions. As an application, we test whether the *ex ante* risk premium is always positive. We report reliable evidence that the *ex ante* risk premium is negative in some states of the world; these states are related to periods of high expected inflation and especially to downward-sloping term structures.

Key words: Ex ante risk premium; Inequality restrictions; Conditional asset pricing models
JEL classification: G12; C22

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*A previous version of the paper was circulated under the title: *Testing Inequality Restrictions Implied by Conditional Asset Pricing Models*. We would like to thank George Constantinides, Jay Shanken (the referee), Don Keim, Ken Singleton, Rob Stambaugh, Robert Whitelaw, and Bill Schwert (the editor) for helpful comments and suggestions. We are also grateful to seminar participants at Columbia University, McGill University, University of Michigan, University of Pennsylvania, University of Rochester, the 1993 AFA meetings in Anaheim and the discussant, Peter Bossaerts, and especially the 1993 Utah Winter Finance Conference and the discussant, Gautam Kaul. Research support from the Stern School of Business and the Finance Department (Boudoukh), the University of Pennsylvania Research Foundation and the Geewax-Terker Research Fund (Richardson), and the Business Associates' Fund (Smith) is gratefully acknowledged. The Fortran code for testing inequality restrictions is available upon request.

1. Introduction

Financial asset pricing theory is focused largely upon relating conditional expected returns to observable economic variables. The corresponding asset pricing models often lead to restrictions on the signs, but not the magnitude, of unobservable parameters. A notable example of this phenomenon relates to one of the oldest and most respected financial models, the dynamic capital asset pricing model (CAPM), in which the *ex ante* risk premium is always positive. The idea is that, if agents are risk-averse and therefore want compensation for risk, the expected return on aggregate wealth (i.e., the market portfolio) should exceed the risk-free rate. While this result is not, in general, a necessary condition for capital market equilibrium, it has widespread support:

> *In estimating models of the expected market return, the non-negativity restriction of the expected excess return should be explicitly included as part of the specification.* [Merton (1980)]

While the empirical literature is filled with anecdotal evidence regarding the positivity of the risk premium, most of the evidence refers to *ex post* fitted estimates of the risk premium. In light of the importance of this condition, the absence of *ex ante* tests related to this particular example, as well as within the overall field of conditional asset pricing with implied inequality restrictions, seems surprising.

There are two main reasons for the lack of formal hypothesis testing of this restriction. First, conditional on a wide array of information, the econometrician faces the restriction that the expected risk premium is always positive. This implies multiple inequality restrictions which are not covered by standard econometric theory. Second, conditional expected returns are unobservable to the econometrician. Hence, previous statements in the literature concerning the positivity restriction on the risk premium rely heavily on the parameterization of an underlying model for expected returns.

This paper provides two major contributions. The first contribution is to extend the methodology for testing inequality constraints, applied recently in finance by Richardson, Richardson, and Smith (1992), to allow moments to be conditioned on observable information.[1] In particular, our procedure takes into account the unobservability of expected returns by employing an instrumental variables approach. The method can be thought of as the natural analogue to existing procedures first studied in finance by Hansen and Singleton (1982) and

[1]The multiple inequality testing problem was first looked at in the statistics literature by Bartholomew (1961), Kudo (1963), and Perlman (1969). More recently, the procedures developed in those papers have been generalized to more extensive problems in econometrics by Gourieroux, Holly, and Monfort (1982), Kodde and Palm (1986), and particularly Wolak (1989a, 1989b, 1991), among others. The use of conditioning information, however, is novel to this paper.

Gibbons and Ferson (1985), but instead applied to an inequality constraints setting. Given the fact that empirical asset pricing is primarily concerned with conditional moments, and that most models imply inequality restrictions, widespread potential applications of our approach to the financial econometrics literature exist.

The second contribution of this paper is empirical in nature, in that new evidence regarding the positivity of the *ex ante* risk premium is developed. Specifically, we provide evidence that, in this multiple inequality restrictions framework, the *ex ante* risk premium is negative in some periods. As an aside, we identify the states of the world in which this violation occurs. While existing research in finance is suggestive of negative *ex ante* risk premiums [e.g., Fama and Schwert (1977) find that fitted values of the premium are sometimes negative], this paper provides the first reliable statistical evidence of negative *ex ante* risk premiums.

The paper is organized as follows. Section 2 describes the test methodology and corresponding statistics for testing whether the expected risk premium is positive. Section 3 applies the procedure to data commonly used in current conditional asset pricing studies. Some concluding remarks are offered in section 4.

2. Test methodology

2.1. The ex ante risk premium

The restriction that the *ex ante* risk premium is nonnegative can be written as[2]

$$\mathrm{E}_t[R_{mt+1}] \geqslant R_{ft}, \tag{1}$$

where R_{mt+1} is the return on the market portfolio from t to $t+1$ and R_{ft} is the return on the risk-free asset from t to $t+1$. The fundamental question is whether this condition is ever violated. Further, if violations take place, why should agents view holding the market portfolio, a nominally risky asset, as advantageous to holding a nominally riskless asset, and hence charge a negative *ex ante* risk premium?

It is possible to show that, in the context of dynamic consumption-based asset pricing models, this result is only possible if the conditional covariance between the marginal rate of substitution and the (excess) return on the market is positive in some states of the world. While it is theoretically possible to obtain this condition, there is some debate regarding its plausibility using reasonable

[2]From a theoretical standpoint, Merton (1982, 1990) provides justification and several sufficient equilibrium conditions for a positive risk premium in both static and dynamic settings.

parameter values [see Tauchen and Hussey (1991) for a discussion in the context of Lucas' (1978) asset pricing model]. Nevertheless, it remains an empirical question as to whether this condition actually occurs.

One of the best-known financial models, the conditional version of the CAPM, implies a nonnegativity restriction, although for the most part this specification has been ignored in testing. For example, Gibbons and Ferson (1985) test implications derived from the linear relation between expected returns and their market beta.[3] Using international data, and in an expanded frame work, Harvey (1991a) tests the conditional CAPM and again concentrates on the linearity relation. However, these and similar approaches require some knowledge (which the econometrician does not have) about the true time-varying movements in expected returns, covariances, and variances of the underlying assets.

Of particular interest to the conditional CAPM, tests of model (1) above represent a possible solution to the critique most commonly associated with Roll (1977).[4] In standard tests of the CAPM, identification of the market portfolio is necessary for an appropriate test. That is, the linear relation holds mathematically as an identity. The only empirical questions are which portfolios satisfy this relation, and, with respect to the CAPM, whether one of these portfolios is the market portfolio. Latent-variable approaches to the Roll critique which avoid identifying the market portfolio, such as Gibbons and Ferson (1985), have been largely unsuccessful [see Wheatley (1989)]. In a related context, Kandel and Stambaugh (1987) and Shanken (1987) manage to place bounds on the correlation of the market proxy and the unobservable market portfolio. However, these bounds are unconditional and imply relatively high *ex ante* correlations [Kandel and Stambaugh (1987) find that the CAPM can be rejected for correlation values over 0.7 in their sample].

If the conditional CAPM is true, then the return on the market proxy R_{pt} must satisfy the following relation:

$$\mathrm{E}_t[(R_{pt+1} - R_{ft})] = \frac{\mathrm{cov}_t[R_{pt+1}, R_{mt+1}]}{\mathrm{var}_t[R_{mt+1}]}\,\mathrm{E}_t[(R_{mt+1} - R_{ft})]\,.$$

Under the null, the true *ex ante* market risk premium must be positive. Therefore, if the conditional covariance between our market proxy and the

[3]For additional examples of similar tests, see Campbell (1987) and Ferson, Foerster, and Keim (1993).

[4]A recent paper by Roll and Ross (1992) elaborates on this critique by arguing that there is little point to testing the CAPM relation because the model 'may be of little use in explaining cross-sectional returns no matter how close the (measured) index is to the efficient frontier unless it is exactly on the frontier' (p. 11). While this reasoning is still controversial, it illustrates the potential problems in testing the linear relation implied by the CAPM.

unobservable market portfolio is positive, the *ex ante* risk premium on our proxy must also be positive. Hence, rejection of this restriction for the proxy necessarily implies a rejection of the CAPM. Since the proxy is generally a well-diversified portfolio of assets, the restriction that it must have positive correlation with the market seems weak, in stark contrast to the high correlation normally required from CAPM tests [see Kandel and Stambaugh (1987)]. The drawback from this approach is that the nonnegativity restriction is not sufficient for the CAPM to be true. That is, the nonnegativity restriction may be valid in much less restrictive environments.

Below, we deduce testable inequality restrictions implied by the nonnegativity of the *ex ante* risk premium.

2.2. Implied inequality restrictions

Define μ_t as the *ex ante* risk premium. Eq. (1) simply states that, conditional on all available information, μ_t is nonnegative. With respect to deriving testable restrictions from model (1), note that the econometrician has much less information available to him than do the economic agents. Let us restrict ourselves to instruments available to the econometrician at time t which are also nonnegative for all t (denoted z_t^+) for reasons which will become clear shortly; such instruments might include the level of nominal interest rates, past volatilities of asset returns, the slope of the term structure when it is upward-sloping, et cetera.[5] These instruments are strictly positive and, based on existing empirical work, provide some information about the *ex ante* risk premium.

Because the set of instruments z_t^+ are nonnegative, multiplying both sides of eq. (1) by z_t^+ will not change the sign. Therefore, under model (1), it is possible to write

$$\mathrm{E}_t[(R_{mt+1} - R_{ft}) \otimes z_t^+] = \mu_t \otimes z_t^+ \geqslant 0\,. \tag{2}$$

Rearranging (2) and applying the law of iterated expectations,

$$\mathrm{E}_t[(R_{mt+1} - R_{ft}) \otimes z_t^+ - \theta_{\mu z^+}] = 0\,, \tag{3}$$

where

$$\theta_{\mu z^+} = \mathrm{E}[\mu_t \otimes z_t^+] \geqslant 0\,. \tag{4}$$

[5]Note that restricting z_t^+ in this way does not necessarily throw away information. For example, any random variable z_t can be separated into two positive variables $z_{1t}^+ = \max(0,\ z_t)$ and $z_{2t}^+ = \max(0,\ -z_t)$, which captures all possible states of the world. Alternatively, z_t can be transformed into a positive variable in such a way as to maintain various properties such as orderings of the relative magnitudes of the realizations of z_t through time.

Eq. (3) provides a set of moment conditions for which the econometrician needs to estimate the vector of parameters, $\theta_{\mu z^+}$. It does not matter that the *ex ante* risk premium μ_t is unobservable because the vector of observables (R_{mt+1}, R_{ft}, z_t^+) is enough to identify $\theta_{\mu z^+}$. Of particular interest, under the null hypothesis given in model (1), $\theta_{\mu z^+} \geqslant 0$. Therefore, model (1) implies inequality restrictions on the sample means of the vector $(R_{mt+1} - R_{ft}) \otimes z_t^+$. Rejection of this restriction necessarily means rejection of model (1).

This approach has several attractive features. First, the econometrician does not require a model for conditional expectations. This is especially important because, for many asset pricing theories, conditional expectations are not explicitly modeled.[6] As it turns out, all that needs to be satisfied are some stationarity and ergodicity assumptions regarding the observable variables. Second, there is a substantial literature in finance identifying possible candidates as instrumental variables (e.g., yields on bonds, dividend yields, et cetera). However, much less is known, either theoretically or empirically, about how these variables enter the model for expected returns. Here, there is no assumed functional form, so this is not a potential problem. Third, the restrictions given in (4) can be tested using the technology developed recently in the inequality testing literature.[7] Of particular importance, these restrictions can be tested

[6]Suppose conditional expectations were modeled; for example, it is common practice to assume that returns are linear in the information set of variables. In this case, the method is to regress the risk premium $R_{mt+1} - R_{ft}$ on a set of predetermined variables, the fitted values being the estimate of the conditional values. Similar to eqs. (3) and (4), what are the corresponding restrictions in this linear expectations framework? Specifically, the inequality restriction can be tested using the following equations:

$$\mathrm{E}\begin{pmatrix} [(R_{mt+1} - R_{ft}) - \beta' z_t] \otimes z_t \\ (\beta' z_t) \otimes z_t^+ - \theta_{\mu z^+} \end{pmatrix} = 0,$$

where

$$\theta_{\mu z^+} = \mathrm{E}[\mu_t \otimes z_t^+] \geqslant 0,$$

z_t = set of predetermined variables chosen by econometrician.

Two problems arise here. First, this procedure works only if conditional expectations are linear in the set of variables z_t *chosen* by the econometrician. In most frameworks, there is no reason to believe this is the case. Second, the parameters β are not specified by the null inequality restriction. Because β needs to be estimated, this can lead to problems in deriving globally valid test statistics which hold for all values of the parameters. The tests, however, can be viewed as local around the true parameters β with a corresponding large sample justification [see Wolak (1989a, 1991)].

[7]The inequality restrictions in (4) are generally weaker than the more standard equality restrictions usually tested in finance and, in particular, in the literature dealing with estimation of the risk premium. This point is not a valid criticism, however, of the testing approach described in this paper. If the model imposes inequality restrictions, and we wish to test these implications of the model, then we must rely on an inequality testing framework. Alternative inequality-based testing methods such as Bonferroni-type procedures are inferior. In particular, the correlation across the estimators (and therefore the correlation across the individual statistics) is ignored in the Bonferroni framework [see Wolak (1989b) for a relevant discussion].

jointly and therefore will take into account any correlation across the mean estimators, $\hat{\theta}_{\mu z^+}$. For example, in evaluating the significance of the estimators, the relevant factors are not only the magnitudes of the estimates but also whether these magnitudes are consistent with the covariance matrix of $\hat{\theta}_{\mu z^+}$. The interpretation which the econometrician can give is very similar to that of individual t-tests versus an F-test.

2.3. Test statistic

In this section, we describe a statistic for testing inequality restrictions implied by the null model in (1), i.e., $\theta_{\mu z^+} \geqslant 0$. With respect to testing inequality constraints, our description most closely follows Wolak (1989a) and Kodde and Palm (1986). Because these papers provide an excellent description of the inequality testing methodology, we provide just a brief discussion as it applies to our problem.

In particular, suppose we have T observations on the risk premium $R_{mt+1} - R_{ft}$ and the N-vector z_t^+. Assume these random variables are stationary and ergodic, with finite variances. Let the variance–covariance matrix of the sample moment vector, $(1/T)\sum_{t=1}^{T}[(R_{mt+1} - R_{ft}) \otimes z_t^+]$, be defined as Ω. This matrix can take quite general forms. In brief, the matrix can account for cross-correlation, autocovariances, or heteroskedasticity in the series.

The restriction given in (3) and (4) can be written as a system of N-moment conditions:

$$
\begin{aligned}
&\mathrm{E}[(R_{mt+1} - R_{ft})z_{1t}^+] = \theta_{\mu z_1^+},\\
&\quad\vdots\\
&\mathrm{E}[(R_{mt+1} - R_{ft})z_{Nt}^+] = \theta_{\mu z_N^+},\\
&\mathrm{H}_0:\quad \theta_{\mu z_i^+} \geqslant 0, \qquad \forall i = 1, \dots, N, \qquad (5)
\end{aligned}
$$

versus

$$\mathrm{H_A}:\quad \theta_{\mu z_i^+} \in R^N.$$

With respect to testing the hypothesis in (5), the first step is to estimate the sample means of the product of the observable variables. In particular,

$$\hat{\theta}_{\mu z_i^+} = \frac{1}{T}\sum_{t=1}^{T}[(R_{mt+1} - R_{ft})z_{it}^+], \qquad \forall i = 1, \dots, N.$$

There is no restriction on the sign of these estimates; that is, they may be negative either because the null is false or because sampling error is present. Of importance to the distributional results to follow, the vector $\hat{\theta}_{\mu z^+}$ is asymptotically normal with mean $\theta_{\mu z^+}$ and variance–covariance matrix Ω. The econometrician does not need to know Ω; all that is required is a consistent estimate, denoted $\hat{\Omega}$.[8]

Under the null restriction in (5), the parameter estimates must be non-negative. Following Perlman (1969) and Wolak (1989a), we can derive estimates under the restriction, by minimizing deviations from the unrestricted model:

$$\min_{\theta_{\mu z^+}} (\hat{\theta}_{\mu z^+} - \theta_{\mu z^+})' \Omega^{-1} (\hat{\theta}_{\mu z^+} - \theta_{\mu z^+}),$$

$$\text{subject to} \quad \theta_{\mu z^+} \geqslant 0 .$$

Let $\hat{\theta}^R_{\mu z^+}$ be the solution to this quadratic program.

Within this framework, a natural statistic for testing the hypothesis in (5) is to test how close the restricted estimates $\hat{\theta}^R_{\mu z^+}$ are to the unrestricted estimates $\hat{\theta}_{\mu z^+}$. Under the null, the difference should be small. In particular, the test statistic is given by

$$W \equiv T(\hat{\theta}^R_{\mu z^+} - \hat{\theta}_{\mu z^+})' \hat{\Omega}^{-1} (\hat{\theta}^R_{\mu z^+} - \hat{\theta}_{\mu z^+}) . \tag{6}$$

With respect to the W-statistic's asymptotic distribution, note that the null hypothesis no longer implies a particular value for $\theta_{\mu z^+}$. Nevertheless, using results in Perlman (1989), it is possible to calculate the distribution of the W-statistic for the least favourable value of the null hypothesis and thus of any size test.[9] Unlike the standard statistics under equality constraints, the W-statistic is now distributed as a weighted sum of chi-squared variables with different degrees of freedom. Specifically, the asymptotic distribution of W is given by

$$\sum_{k=0}^{N} \Pr[\chi_k^2 \geqslant c]\, w\!\left(N, N-k, \frac{\Omega}{T}\right), \tag{7}$$

[8]See Perlman (1969) or Wolak (1989a) for relevant proofs. Note that $\hat{\Omega}$ can be estimated using various techniques such as White (1980), Hansen (1982), and Newey and West (1987), among others.

[9]Note that, in general, Ω may depend upon the parameter vector $\theta_{\mu z^+}$. As shown by example in Wolak (1991), when Ω depends on the parameter vector $\theta_{\mu z^+}$ even if $\theta_{\mu z^+} = 0$ is the unique value which satisfies all of the inequalities with equality [as in eq. (5)], the vertex of the positive orthant may not be the least favorable value of $\theta_{\mu z^+}$ for the purposes of constructing the asymptotically exact critical value for the inequality constraints hypothesis test. It is important to point out that the dependence of Ω on $\theta_{\mu z^+}$ (if this dependence in fact exists) does not necessarily pose a problem. Without specifying the estimation further, however, it is not possible to determine the problematic cases. In this instance, it may be appropriate here to treat the inequality hypothesis test as local to the point $\theta_{\mu z^+} = 0$ in the manner described in Wolak (1989a).

where $c \in R^+$ is the critical value for a given size, the weight $w(N, N-k, \Omega/T)$ is the probability that $\hat{\theta}^R_{\mu z^+}$ has exactly $N-k$ positive elements, and χ^2_k for $k=0$ is just a point mass at the origin.

Calculating the weights $w(N, N-k, \Omega/T)$ in (7) can be nontrivial because the weights require evaluation of N-multiple integrals, and closed forms have been calculated for only a small number of restrictions ($N \leqslant 5$) [see Kudo (1963) for exact calculations]. As a partial solution to this problem, Kodde and Palm (1986, table 1, p. 1246) provide upper- and lower-bound critical values which do not require calculation of the weights. These bounds are given by

$$\alpha_l = \tfrac{1}{2}\Pr(\chi^2_1 \geqslant c_l),$$

$$\alpha_u = \tfrac{1}{2}\Pr(\chi^2_{N-1} \geqslant c_u) + \tfrac{1}{2}\Pr(\chi^2_N \geqslant c_u),$$

where c_l and c_u are the lower and upper bounds, respectively, for the critical values of the test.

Only for values in between these bounds does the econometrician have to calculate the weights. For such values, Wolak (1989b) describes a method for calculating the weights based on a Monte Carlo simulation. He suggests that the econometrician simulate a multivariate normal distribution with mean zero and covariance matrix (Ω/T) (denote the vector of realizations from each replication by $\theta^*_{\mu z^+}$). Given the realizations $\theta^*_{\mu z^+}$, the idea is to find the $\hat{\theta}_{\mu z^+}$ which solve the minimization problem

$$\min_{\tilde{\theta}_{\mu z^+}} (\theta^*_{\mu z^+} - \tilde{\theta}_{\mu z^+})(\hat{\Omega}/T)^{-1}(\theta^*_{\mu z^+} - \tilde{\theta}_{\mu z^+}),$$

$$\text{subject to} \quad \tilde{\theta}_{\mu z^+} \geqslant 0.$$

As Wolak (1989b) points out, the approximate weight $\hat{w}(N, N-k, \hat{\Omega}/T)$ will be the fraction of replications in which $\hat{\tilde{\theta}}_{\mu z^+}$ has exactly $N-k$ elements exceeding zero.

Note that the above results are only valid asymptotically. However, there do exist some small-sample results for inequality constraint-testing procedures. But, just as in testing equality constraints, these results hold only under very strong assumptions. For example, in the general linear model, under the assumption that the errors are normally distributed and under restrictions on the error's (possibly unknown) covariance matrix, Wolak (1987) shows that the Wald statistic [in (6)] and analogous Lagrange multiplier and likelihood ratio statistics are distributed as a weighted sum of F-distributions. If the covariance matrix takes on more general forms, then orderings in these statistics along the lines of equality tests [e.g., as described in Berndt and Savin (1976)] still carry through. Outside of these results, little is generally known about the small-sample properties of inequality constraint-testing procedures, but that is true for equality constraints tests as well.

In the more general environment of this paper, it is important to recognize that our application of the statistic and its corresponding asymptotic distribution is subject to the usual small-sample caveats. Some additional observations, therefore, are in order. First, we conjecture that, if anything, small-sample issues may be less of a concern here than they normally are for testing equality constraints. Under an equality restriction, biases in the estimators can produce severe deviations from the null in small samples. An inequality restriction, however, is much weaker; hence, biased estimators may fall within the viable range of the inequality-based null hypothesis. Second, and a related point, we described above the asymptotic distribution of W for the least favorable value (and, therefore, any size test) of our null hypothesis. Thus, for many parameter values, the cutoff value is conservative and small-sample issues may be less important. Third, it would be interesting to better understand how well inequality testing procedures perform in small samples with respect to the local versus global nature of the test (e.g., see footnote 9) and the Kodde and Palm (1986) bounds described above. One possible way to examine this issue would be to apply a Monte Carlo analysis in this framework. Unlike testing equality constraints in which the null is specified *a priori*, however, it is necessary here to specify how 'well' the parameters satisfy the constraint under the inequality-based null. Thus, the parameterization in the Monte Carlo study takes on even more importance. We hope to explore some of these issues in future research.

As a final comment on the statistical procedure, suppose the null hypothesis $\theta_{\mu z^+} \geqslant 0$ is satisfied. It may be of interest to determine the extent to which $\theta_{\mu z^+}$ is greater than instead of equal to zero. Gourieroux, Holly, and Monfort (1982) derive tests for the case of testing a null $\theta_{\mu z^+} = 0$ versus the alternative $\theta_{\mu z^+} \geqslant 0$. This is an especially useful procedure for testing equality constraints when the econometrician can restrict $\theta_{\mu z^+}$ to a particular sign or range of values under interesting alternative models. Resorting back to an equality testing framework here would lead to much less power. In particular, Yancey, Bohrer, and Judge (1982) show that the power is greater with the Gourieroux et al. one-sided test of equality versus the usual two-sided tests.

3. Results

A substantial literature has emerged in finance documenting time-varying properties of expected returns [see Fama (1991) and Hawaweni and Keim (1992) for extensive surveys]. The important stylized facts from this literature can be summarized as follows:

- There exists a negative relation between expected stock returns and T-bill rates. This relation can be described partially by the anomalous negative relation between stock returns and expected inflation [e.g., Fama and Schwert (1977)] and, to some extent, by expectations about the future real rate at different stages of the business cycle [e.g., Harvey (1988)].[10]

- Expected returns on the market are, for the most part, an increasing function of risk, as measured by the volatility of the market return [e.g., see Merton (1990) for a discussion of sufficient conditions for this result]. Empirical evidence tends to support this result [e.g., Merton (1980), French, Schwert, and Stambaugh (1987), and Harvey (1991b)], although this is not a universally accepted conclusion [see Pagan and Hong (1991)].

- The market's dividend yield [e.g., Fama and French (1988, 1989)], the slope of the term structure of interest rates [e.g., Campbell (1987), Fama and French (1989), and Chen (1991)], and the spread between portfolios of investment grade and noninvestment grade bonds [e.g., Keim and Stambaugh (1986) and Fama and French (1989)] also contain information about expected returns. The conclusion is that these variables are positively related to expected returns; in particular, they capture movements in business and default risk through time.

What is the implication of these stylized facts with respect to negative *ex ante* risk premiums?

With respect to the positivity of the risk premium, Fama and Schwert (1977), Harvey (1991b), and Whitelaw (1992), among others, report some negative fitted estimates of the expected risk premium. Because these fitted values are *ex post* estimates, however, it is difficult to distinguish between sampling error and a true negative *ex ante* risk premium. In fact, most researchers argue that the negative expected risk premiums are most probably sampling error [e.g., see Fama and Schwert (1977)]. Despite the aforementioned volume of research, and the importance of the nonnegativity restriction on the *ex ante* risk premium, it may seem surprising that no formal tests of this restriction have taken place. Because the restriction implies a set of conditional inequality constraints, however, the test requires the methodology introduced in this paper.

[10] Note that Merton (1980) takes a somewhat different view of the estimates of the *ex ante* risk premium in periods of high T-bill rates. He argues that the *ex post* fitted values are poor estimates because the estimation has not explicitly incorporated the nonnegativity of the *ex ante* risk premium. We avoid Merton's criticism because our test is *ex ante*; in particular, our approach treats Merton's null hypothesis of a positive *ex ante* risk premium as given and then tests this hypothesis directly.

3.1. Empirical tests: A first look

Given the numerous studies using post-World War II data on stock returns and interest rates, there are potential data-snooping biases. It seems worthwhile then to study a long time series, which is less subject to these criticisms. Schwert (1990) and Siegel (1992) have developed data sets on stock returns, inflation, and bond yields covering the past two centuries. There are several advantages to employing this data in testing the nonnegativity restriction on the *ex ante* risk premium. First, given the rarity of some economic events (such as downward-sloping yield curves), the long time series hopefully affords us enough periods to extract information. Second, given the paucity of cross-sectional asset returns data over this period, our procedure is also the only viable approach to testing the conditional CAPM over this long term. Third, there has been considerable research trying to explain the large unconditional average risk premium. Recent work by Siegel (1992) argues that the magnitude of the risk premium is sample-specific. It seems reasonable therefore to study the risk premium in a conditional asset pricing framework over all the sample periods in question.

We view our tests as a reinterpretation of existing evidence for its effect on the nonnegativity restriction of the *ex ante* risk premium. As our choice of instruments, therefore, we condition on existing research. We do not take a position on the potential data-snooping biases which arise by choosing instruments based on previous research. We simply take the existing evidence as given, although we should note that much of our sample period does not overlap with the periods used in previous studies. In particular, in the post-World War II period, the above stylized facts suggest that the risk premium is negatively related to T-bill rates, yet positively related to volatility, the dividend yield, and the slope of the term structure. Therefore, to reflect potential negative *ex ante* risk premiums, we choose instruments, z_t^+, which reflect periods of implied low risk premiums.

For example, with respect to the risk-free rate, we condition on times when R_{ft} is high.[11] In particular, define $z_{1t}^* = 1$ if the risk-free rate is high, and $z_{1t}^* = 0$ otherwise. For normalization purposes, we use the instrument $z_{1t}^+ = 1/\mathrm{E}[z_{1t}^*]$ if the risk-free rate is high, and $z_{1t}^+ = 0$ otherwise. In this paper, we define the risk-free rate as *high* when it lies above its long-run mean.[12] Similarly, for the other three instruments, $z_{2t}^* = 1$ if the term structure is downward sloping (i.e., $\Delta r_{lf,t} < 0$, where $\Delta r_{lf,t}$ is the spread between the long and short rate of interest);

[11] To coincide with Fama and Schwert (1977) and their analysis of the Fisher model, high risk-free rates can be associated with high expected inflation. It should be noted, however, that outside their sample period, there is some evidence (not entirely uncontroversial) that the Fisher model for interest rates does not hold [see, for example, Barsky (1987)]. In terms of the statistical analysis in this paper, what matters is that R_{ft} is *ex ante* and has reasonable distributional properties. Whether this variable captures expected inflation ralates to our economic interpretation of the results and not the inequality testing procedure *per se*.

[12] Under the necessary assumptions of stationarity and a large sample and the assumption that the market knows the distribution of the z_t, the analysis carries through to this setting.

Table 1

Tests of whether the *ex ante* risk premium is always positive.

The sample covered in this study include annual data on aggregate U.S. stock returns, inflation, long and short rates of interest, and dividend yields over three time periods (1802–1990, 1802–1896, and 1897–1990). A complete description of the data are provided by Schwert (1990) and Siegel (1992). The statistic W tests whether the *ex ante* risk premium is positive. Specifically, W is a joint test of multiple inequality restrictions corresponding to high T-bill rates (e.g., high expected inflation states), downward-sloping term structures, low-volatility periods, and low-dividend-yield periods (only available for latter subperiod). The estimators, $\hat{\theta}_{\mu z_i^+}$, represent the conditional mean of the risk premium in these states. Also given are the probability of these states and the standard errors of the conditional means. Note that high (low) is defined as being above (below) the long-run mean of the variables (i.e., expected inflation, volatility, et cetera). Note that the numbers are annualized and reported in decimal terms. All estimates are adjusted for conditional heteroskedasticity and serial correlation using the method of Newey and West (1987).

Statistic	1802–1990	1802–1896	1897–1990
High expected inflation			
Probability of state	0.495	0.500	0.457
Mean $\hat{\theta}_{\mu z_i}$	− 0.0123	− 0.0186	− 0.0064
(Standard error)	(0.0148)	(0.0241)	(0.0203)
Downward-sloping yield curve			
Probability of state	0.388	0.511	0.[illegible]66
Mean $\hat{\theta}_{\mu z_i}$	− 0.0285	− 0.0212	− 0.0425
(Standard error)	(0.0191)	(0.0233)	(0.0332)
Low volatility			
Probability of state	0.702	0.670	0.734
Mean $\hat{\theta}_{\mu z_i}$	0.0216	− 0.0077	0.0529
(Standard error)	(0.0157)	(0.0129)	(0.0223)
Low dividend yield			
Probability of state	NA	NA	0.564
Mean $\hat{\theta}_{\mu z_i}$	NA	NA	0.014[illegible]
(Standard error)	NA	NA	(0.028[illegible])
Multiple inequality restrictions statistic W	2.229	0.852	1.638
(p-value)	(0.1388)	(0.3040)	(0.3068)

$z_{3t}^* = 1$ if the volatility of the risk premium, σ_{mt}, is low; $z_{4t}^* = 1$ if the dividend yield of the market, D_{mt}/P_{mt}, is low; and $z_{it}^* = 0$ otherwise, for all i.[13] Note that these instruments are normalized similar to the risk-free rate above.

These normalized instruments have a clear economic interpretation. Consider the instrument associated with high T-bill rates, z_{1t}^+, and its corresponding estimator from section 2.3, $\hat{\theta}_{\mu z_1^+} = (1/T)\sum_{t=1}^{T} [(R_{mt+1} - R_{ft})z_{1t}^+]$. Specifically, $\hat{\theta}_{\mu z_1^+}$ is the sample mean of the risk premium, *conditional on high T-bill rates*. For each instrument, table 1 provides the conditional means of the risk premium and

[13]The volatility of the risk premium is estimated from the previous twelve months of monthly data according to the procedure in French, Schwert, and Stambaugh (1987). Note that data on dividends are only available after 1870 [Schwert (1990)], and hence this instrument is included only in the second subperiod.

the estimated probability associated with each conditioning event. For example, consider the overall period 1802–1990. The annual means of the risk premium are -1.23%, -2.85%, and 2.16% conditional on either high T-bill rates, a downward-sloping term structure, or low volatility, respectively. Of some interest, note that these states occur quite frequently; in particular, high T-bill rates and downward-sloping term structures occur 49.5% and 38.8% of the time, respectively.[14] In contrast, given that the unconditional risk premium equals 3.1% over the sample, the 2.16% conditional mean in low volatility periods suggests that low volatility has little impact on the level of the risk premium.

Of course, these results are only suggestive of negative *ex ante* risk premiums. Under Merton's (1980) and others' null hypothesis that the *ex ante* risk premium is positive, it is important to formally test this restriction. Applying the procedure in section 2, the multiple inequality restrictions statistic derived in eq. (6) is 2.23, which represents a p-value of 0.139. At first glance, this result may seem surprising because $\hat{\theta}_{\mu z_2^+}$, by itself, is significant at the 10% one-sided level and $\hat{\theta}_{\mu z_1^+}$ is also negative. However, these estimators are correlated; in particular, downward-sloping term structures and periods of high T-bill rates are not independent events. One view of this evidence, then, is that the *ex ante* risk premium is positive, and that the conditional means, $\hat{\theta}_{\mu z_1^+}$ and $\hat{\theta}_{\mu z_2^+}$, are picking up similar sampling error.

The subperiods 1802–1896 and 1897–1990 provide results similar to the overall sample results. In particular, both high T-bill rates and downward-sloping term structures imply negative *ex ante* risk premiums, while low volatility and low dividend yields do not (although the premium for low volatility periods is negative during the 1802–1896 sample). For example, consider downward-sloping term structures during these two subperiods. The conditional mean of the risk premium is -2.1% in the earlier subperiod and -4.25% in the latter subperiod! However, this evidence is not supported by the multiple inequality restrictions test. Specifically, the multiple inequality restrictions statistic is 0.852 (with a p-value equal to 0.304) and 1.638 (with a p-value equal to 0.306) in the earlier and latter subperiods, respectively.

While the joint statistic does not reject the hypothesis that the *ex ante* risk premium is positive, the estimated negative conditional means in both subperiods (for the same instruments) provide some independent evidence (albeit weak) that the positivity restriction may be violated. One problem with the

[14]This does not imply that the *ex ante* risk premium is always negative when T-bill rates are high, that is, 49.5% of the time. Note that the conditional mean $\hat{\theta}_{\mu z_1^+}$ is an average over the *ex ante* risk premiums in high T-bill rate periods. Thus, given the negative $\hat{\theta}_{\mu z_1^+}$, all the econometrician knows is that there exist some states of T-bill rates in which the *ex ante* risk premium is negative. However, the negative means suggest the *ex ante* risk premium is either negative frequently in these states or large and negative in only some of these states.

above analysis is that potentially important information has been discarded in evaluating the conditional means. Specifically, the magnitude of the T-bill rate (when it is high), the slope of the term structure (when it is downward-sloping), the estimate of volatility (when it is low), and so forth, have all been ignored in estimation. Below, we address this issue in more detail.

3.2. Empirical tests: A closer look

As mentioned above, note that the analysis so far has not utilized all of the available information. For example, z_{1t}^{+} above transforms the T-bill instrument to a 1/0 variable, depending on whether T-bill rates are high or low. To the extent that the magnitude of the T-bill rate has some relation to the *ex ante* risk premium, we should incorporate this relation in our test procedure. One way to do this is to use the instrument $z_{1t}^{*} = \max(0, R_{ft} - \mathrm{E}[R_{ft}])$, which conditions on both periods with high rates and the magnitude of those rates. If we normalize the instrument such that $z_{1t}^{+} = \max(0, R_{ft} - \mathrm{E}[R_{ft}])/\mathrm{E}[z_{1t}^{*}]$, the estimator $\hat{\theta}_{\mu z_1^*}$ again has a clear economic interpretation. In particular, $\hat{\theta}_{\mu z_1^*}$ equals the conditional mean of the risk premium, weighted most by high T-bill rates.

For each of the other predetermined variables, we similarly define an informative instrument: $z_{2t}^{*} = \max(0, -\Delta r_{lf,t})$, $z_{3t}^{*} = 1/\sigma_{mt}$, and $z_{4t}^{*} = P_{mt}/D_{mt}$. These instruments are also all normalized for purposes of economic interpretation. To get a feel for the weights, figs. 1–3 graph the *ex post* risk premium and corresponding weights for T-bills (e.g., expected inflation), downward-sloping term structures, and low volatility respectively, over the sample period 1802–1990. Several characteristics of these weights are particularly interesting. First, in fig. 1, it is clear that downward-sloping term structures forecast negative future risk premiums. Although substantial weight is placed on the earlier sample period, the large drops in returns in the early 1900's, the 1930's, and the 1970's correspond to downward-sloping term structures and hence positive weights. Second, in fig. 2, high T-bill rates forecast negative risk premiums similar to downward-sloping term structures, especially in the earlier period. However, the fall in the risk premium in the early 1900's and 1930's is completely missed. Note that this instrument places its greatest weight on the 1970's and early 1980's, and successfully captures the negative risk permium which occured during this period. Finally, in fig. 3, the low volatility instrument places large weight on the earlier period; however, it does not appear to be strongly related to the drops in the risk premium that actually occurred. Throughout the rest of the sample period (albeit with small weights), low volatility appears to be associated with negative premiums in the middle of the sample (i.e., 1870–1920) but nowhere else. Consistent with the results below, however, volatility apparently contains little information about negative risk premium states.

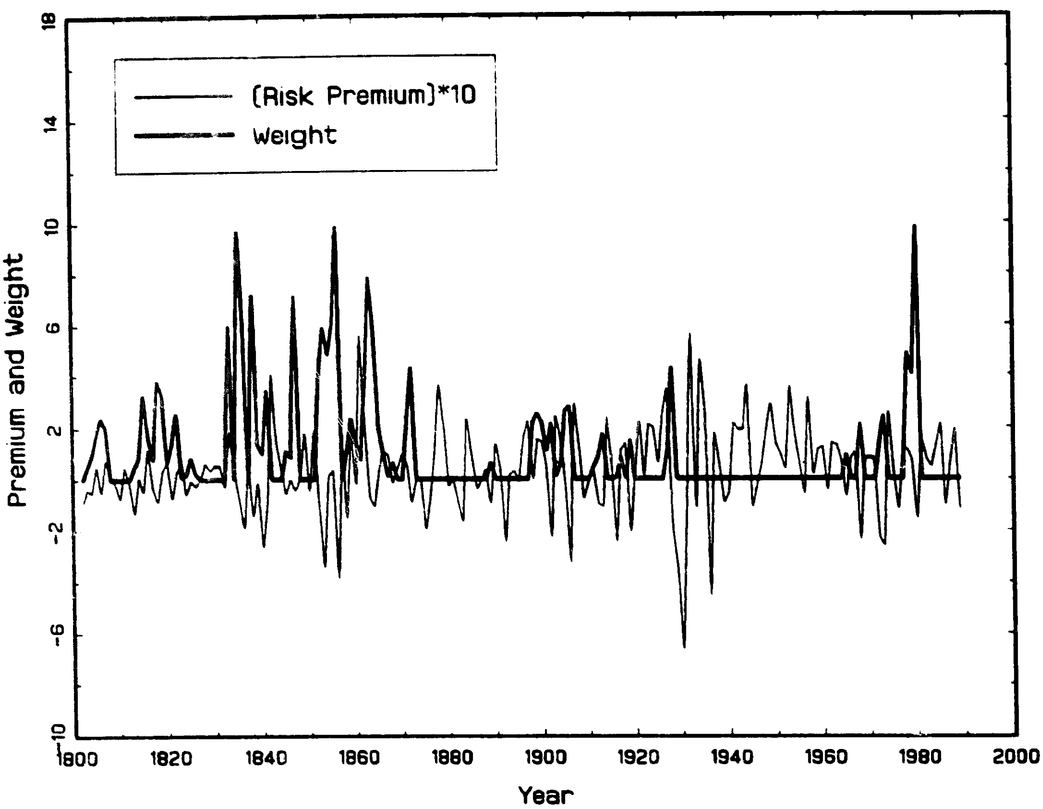

Fig. 1. Annual risk premium and weights corresponding to periods of downward-sloping term structures in the period 1802–1990. The weights are given by $z_{2t}^* = \max(0, -\Delta r_{lf,t})/E[z_{2t}^*]$, where $\Delta r_{lf,t}$ is the spread between long and short rates of interest and $E[z_{2t}^*] = E[\max(0, -\Delta r_{lf,t})$ is the normalization factor. These weights are used to estimate conditional means of the risk premium over the sample period 1802–1990 (see table 2). Note that the weights are chosen *ex ante* with the intention of capturing periods of low conditional expected risk premiums.

In terms of the empirical tests, table 2 provides the conditional means of the risk premium, weighted by the magnitudes of each instrumental variable. The results are stronger than those given in table 1. For example, over the sample period 1802–1990, the annual means of the risk premium are -1.53%, -6.85%, and -0.22%, conditionally weighted on either high T-bill rates, a downward-sloping term structure, or low volatility, respectively. This evidence is consistent with the *ex ante* risk premium being related to both the sign and magnitude of the instrumental variables. In particular, when we condition on the magnitude of the downward slope of the term structure, the decrease in the conditional expected risk premium from -2.85% to -6.85% suggests that the magnitude of the term structure spread has especially relevant information for expected returns.

In contrast to the 1/0 instruments in section 3.1, the more informative instruments lead to a rejection of the *ex ante* risk premium being positive. For example, the multiple inequality restrictions statistic is 5.156, which represents

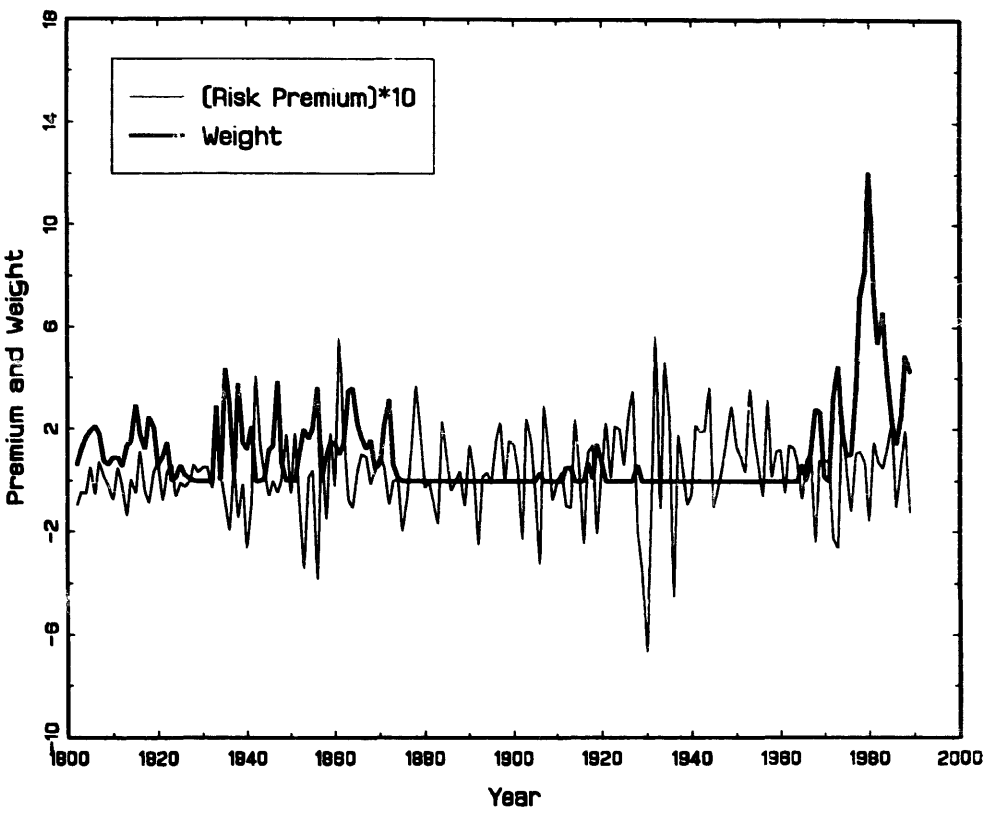

Fig. 2. Annual risk premium and weights corresponding to periods of high T-bill rates (relative to its long-run mean) in the period 1802–1990. The weights are given by $z^{+}_{1t} = \max(0, R_{ft} - \mathrm{E}[R_{ft}])/\mathrm{E}[z^{*}_{1t}]$, where R_{ft} is the T-bill rate and $\mathrm{E}[z^{*}_{1t}] = \mathrm{E}[\max(0, R_{ft} - \mathrm{E}[R_{ft}])]$ is the normalization factor. These weights are used to estimate conditional means of the risk premium over the sample period 1802–1990 (see table 2). Note that the weights are chosen *ex ante* with the intention of capturing periods of low conditional expected risk premiums.

a p-value at most equal to 0.032. While all the weighted conditional means are negative, the strongest evidence again relates to downward-sloping term structures and, to a lesser extent, high T-bill rates. One possible explanation of this result is data error, especially in the pre-Civil War period.[15]. To check this, we also performed our tests for each subperiod, 1802–1896 and 1897–1990.

In both subperiods, the multiple inequality restrictions statistic suggests rejection of the positivity of the *ex ante* risk premium. Specifically, the joint statistic equals 4.275 (with corresponding p-value 0.051) and 4.365 (with corresponding p-value 0.070) in the earlier and latter samples, respectively. Of some interest, the determining states again are downward-sloping term structures,

[15]At least with the stock return data, however, we might expect the bias to actually go in the opposite direction. The pre-Civil War stock index has a survivorship bias to the extent that it does not contain poorly-performing stocks during this period. We are grateful to Bill Schwert for pointing this out to us.

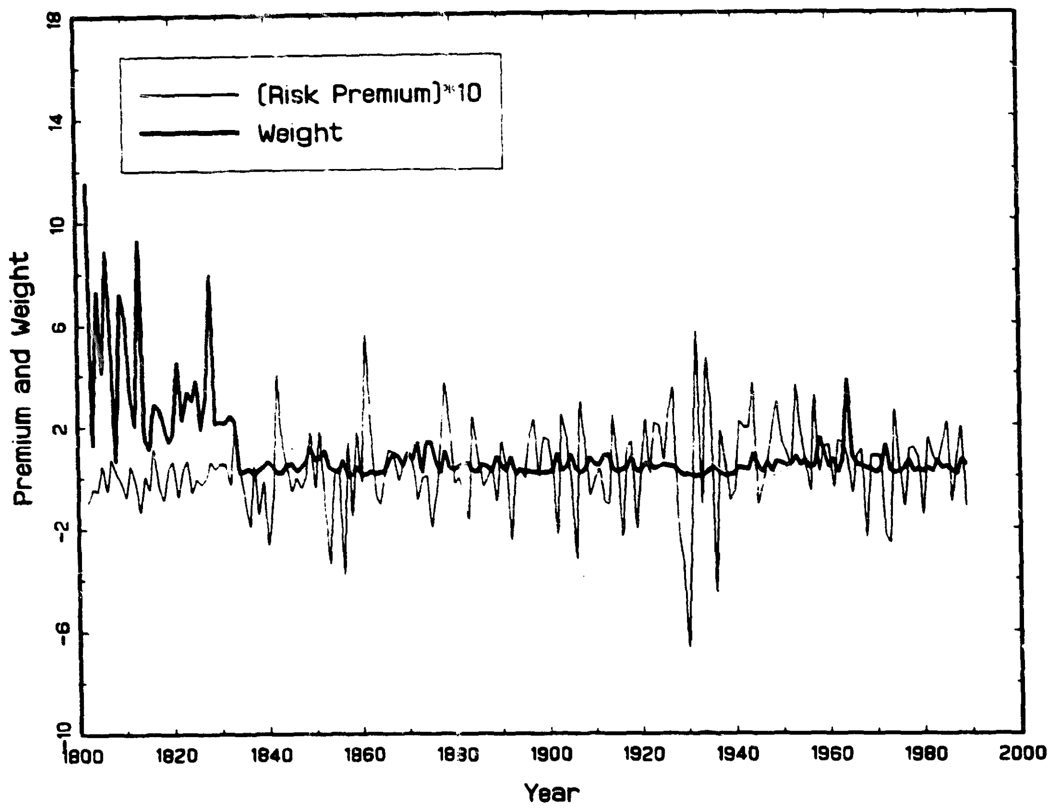

Fig. 3. Annual risk premium and weights corresponding to periods of low volatility in the period 1802–1990. The weights are given by $z_{3t}^{+} = \sigma_{mt}^{-1}/\mathrm{E}[z_{3t}^{*}]$, where σ_{mt} is the conditional volatility of the market return at year t (estimated over the previous 12 months) and $\mathrm{E}[z_{3t}^{*}] = \mathrm{E}[1/\sigma_{mt}]$ is the normalization factor. These weights are used to estimate conditional means of the risk premium over the sample period 1802–1990 (see table 2). Note that the weights are chosen *ex ante* with the intention of capturing periods of low conditional expected risk premiums.

with weighted conditional means of − 6.71% and − 7.20%, respectively, in each subperiod. In contrast to section 3.1, the conditional mean for high T-bill rates switches sign from the earlier sample to the latter sample period. In particular, the conditional mean increases from − 5.17% to 0.42%. Similarly, the mean associated with low volatility states increases from − 1.66% to 5.38%. Taking the point estimates as given, this evidence suggests that negative *ex ante* risk premium are more likely and of larger magnitude in the nineteenth century. This conclusion may be related to developing markets, in particular the possibility of more extreme business cycles during these periods, or perhaps it is just an artifact of the data. Nevertheless, in all the periods (i.e., 1802–1990, 1802–1896, and 1897–1990), there is statistical evidence that the *ex ante* risk premium can be negative. Moreover, our choice of instruments coincides with those most commonly used in current empirical analyses of the risk premium.

Table 2

Tests of whether the *ex ante* risk premium is always positive.

The sample covered in this study include annual data on aggregate U.S. stock returns, inflation, long and short rates of interest, and dividend yields over three time periods (1802–1990, 1802–1896, and 1897–1990). A complete description of the data is provided by Schwert (1990) and Siegel (1992). In contrast to table 1, the statistic explicitly incorporates the magnitude of these variables. Specifically, the test conditions on the magnitude of expected inflation in high expected inflation states, the slope of the term structure if downward-sloping, the inverse of volatility, and the inverse of the dividend yield. The estimators, $\hat{\theta}_{\mu z_i^+}$, represent the conditional mean of the risk premium in these states, weighted by the magnitude of the instrumental variables. Also provided are the standard errors of the conditional means. Note that the numbers are annualized and reported in decimal terms. All estimates are adjusted for conditional heteroskedasticity and serial correlation using the method of Newey and West (1987).

Statistic	1802–1990	1802–1896	1897–1990
High expected inflation			
Weighted mean $\hat{\theta}_{\mu z_i}$	−0.0153	−0.0517	0.0042
(Standard error)	(0.0192)	(0.0273)	(0.0271)
Downward-sloping yield curve			
Weighted mean $\hat{\theta}_{\mu z_i}$	−0.0685	−0.0671	−0.0720
(Standard error)	(0.0302)	(0.0390)	(0.0345)
Inverse of volatility			
Weighted mean $\hat{\theta}_{\mu z_i}$	−0.0022	−0.0166	0.538
(Standard error)	(0.0135)	(0.0151)	(0.0229)
Inverse of dividend yield			
Weighted mean $\hat{\theta}_{\mu z_i}$	NA	NA	0.0427
(Standard error)	NA	NA	(0.0171)
Multiple inequality restrictions statistic W	5.156	4.275	4.365
(p-value)	(0.0316)	(0.0505)	(0.0696)

4. Conclusion

Using recently developed techniques for testing inequality constraints, this paper provides a new methodology for testing restrictions implied by conditional asset pricing models. Of particular interest, this methodology is easy to implement, requires little knowledge of the conditional distribution of asset returns, and is valid under fairly weak assumptions. As an important application of this method, we investigate the nonnegativity restriction on the *ex ante* risk premium. Our results suggest that the *ex ante* risk premium is negative in some states of the world. These states appear to be related to periods of high T-bill rates and especially to times in which the term structure is downward-sloping. From a consumption-based asset pricing perspective, the implication of this result is that the conditional covariance between the marginal rate of substitution and the (excess) return on the market is positive during these periods. We hope to explore this stylized observation in future research.

On a more cautionary note, the test methodology described in section 2 provides the econometrician with considerable flexibility in choosing instrumental variables in the information set. The choice of these variables should come from *a priori* theory or economic intuition. The tendency to pick 'powerful' variables *ex post* can lead to incorrect application of the statistics. Note that this is *true of all* conditional asset pricing tests involving instrumental variables – it is not unique to this paper. In the context of current empirical work in finance, our procedure shows how to apply existing econometric method to testing inequality restrictions implied by conditional asset pricing models. The econometric techniques developed in this paper thus seem especially suited to financial asset pricing models. As illustrations, corporate finance theories [e.g., mean and variance effects around event dates], the financial anomaly literature [e.g., the *size*, *book-to-market*, and *price-to-earnings* effects described in Fama and French (1992), among others], term structure models [e.g., the *liquidity preference hypothesis* studied in Fama (1986)], the volatility bounds literature [surveyed in Gilles and LeRoy (1991)], dynamic asset pricing restrictions [implied by Hansen and Jagannathan (1991)], and stochastic Euler equations in the presence of transactions costs [e.g., see He and Modest (1992)] all imply multiple inequality restrictions in a conditional asset pricing framework.[16]

References

Barsky, Robert J., 1987, The Fisher hypothesis and the forecastability and persistence of inflation, Journal of Monetary Economics 19, 3–24.

Bartholomew, D.J., 1961, A set of homogeneity of means under restricted alternatives, Journal of Royal Statistical Society Series B 23, 239–281.

Berndt, Ernst R. and N. Eugene Savin, 1976, Conflict among criteria for testing hypotheses in the multivariate linear regresion model, Econometrica 45, 1263–1277.

[16]In our application, note that inequality constraints are the appropriate null hypothesis. In other applications, such as the financial anomaly or corporate finance literature, it may be more reasonable to treat the theories as alternative hypotheses. The inequality constraints testing methodology is still the appropriate procedure in this case. For example, as described briefly in section 2.3, Gourieroux, Holly, and Monfort (1982) describe how to test a null $\theta_{\mu z^*} = 0$ versus the alternative $\theta_{\mu z^*} \geqslant 0$. In particular, it is possible to show that an analogous test statistic to (6) for this hypothesis is

$$T(\hat{\theta}^R_{\mu z^*})'\hat{\Omega}^{-1}\hat{\theta}^R_{\mu z^*},$$

with corresponding asymptotic distribution for the least favorable value of the null hypothesis,

$$\sum_{k=0}^{N} \Pr[\chi^2_k \geqslant c]\, w(N,k,\Omega/T).$$

Thus, tests of financial models against alternative theories should be performed in the inequality testing framework.

Campbell, John Y., 1987, Stock returns and the term structure, Journal of Financial Economics 18, 373-399.
Chen, Nai-Fu, 1991, Financial investment opportunities and the macroeconomy, Journal of Finance 46, 529-554.
Fama, Eugene F., 1986, Term premiums and default premiums in money markets, Journal of Financial Eonomics 17, 175-196.
Fama, Eugene F., 1991, Efficient capital markets: II, Journal of Finance 46, 1575-1617.
Fama, Eugene F. and Kenneth R. French, 1988, Dividend yields and expected stock returns, Journal of Financial Economics 22, 3-25.
Fama, Eugene F. and Kenneth R. French, 1989, Business conditions and expected returns on stocks and bonds, Journal of Financial Economics 25, 23-49.
Fama, Eugene F. and Kenneth R. French, 1992, The cross section of expected stock returns, Journal of Finance 47, 427-465.
Fama, Eugene F. and G. William Schwert, 1977, Asset returns and inflation, Journal of Financial Economics 5, 115-146.
Ferson, Wayne E., Stephen R. Foerster, and Donald B. Keim, 1993, General tests of latent variable models and mean-variance spanning, Journal of Finance 48, 131-156.
French, Kenneth R., G. William Schwert, and Robert F. Stambaugh, 1987, Expected stock returns and volatility, Journal of Financial Economics 19, 3-29.
Gibbons, Michael R. and Wayne E. Ferson, 1985, Testing asset pricing models with changing expectations and an unobservable market portfolio, Journal of Financial Economics 14, 217-236.
Gilles, Christian and Stephen F. LeRoy, 1991, Econometric aspects of the variance-bounds tests: A survey, Review of Financial Studies 4, 753-791.
Gourieroux, Christian, Alberto Holly, and Alain Monfort, 1982, Likelihood ratio test, Wald test, and Kuhn-Tucker test in linear models with inequality constraints on the regression parameters, Econometria 50, 63-80.
Hansen, Lars P., 1982, Large sample properties of generalized method of moments estimators, Econometrica 50, 1029-1054.
Hansen, Lars P. and Ravi Jagannathan, 1991, Implications of security market data for models of dynamic economies, Journal of Political Economy 99, 225-262.
Hansen, Lars P. and Kenneth J. Singleton, 1982, Generalized instrumental variables estimation of nonlinear rational expectations models, Econometrica 50, 1269-1286 (with correction in Econometrica 52, 267-268).
Harvey, Campbell R., 1988, The real term structure and consumption growth, Journal of Financial Economics 22, 305-334.
Harvey, Campbell R., 1991a, The world price of covariance risk, Journal of Finance 46, 111-157.
Harvey, Campbell R., 1991b, The specification of conditional expectations, Working paper (Duke University, Durham, NC).
Hawaweni, Gabriel and Donald B. Keim, 1992, On the predictability of common stock returns: World-wide evidence, in: Robert A. Jarrow, William T. Ziemba, and Vojislav Maksimovich, eds., Finance, Handbook series (North-Holland, Amsterdam), forthcoming.
He, Hua and David M. Modest, 1992, Market frictions and consumption-based asset pricing, Working paper (Walter Haas School of Business, University of California, Berkeley, CA).
Kandel, Shmuel and Robert F. Stambaugh, 1987, A mean-variance framework for tests of asset pricing models, Review of Financial Studies 2, 125-156.
Keim, Donald B. and Robert F. Stambaugh, 1986, Predicting returns in the stock and bond markets, Journal of Financial Economics 17, 357-390.
Kodde, David A. and Franz C. Palm, 1986, Wald criterion for jointly testing equality and inequality restrictions, Econometrica 54, 1243-1248.
Kudo, Akio, 1963, A multivariate analogue of the one-sided test, Biometrica 50, 403-418.
Lucas, Robert E., 1978, Asset pricing in an exchange economy, Econometrica 16, 1429-1446.
Merton, Robert C., 1980, On estimating the expected return on the market: An exploratory investigation, Journal of Financial Economics 8, 323-361.
Merton, Robert C., 1982, On the microeconomic theory of investment under uncertainty, in: Kenneth J. Arrow and Michael D. Intriligator, e , Handbook of mathematical economics, Vol. II (North-Holland, Amsterdam).

Merton, Robert C., 1990, Capital market theory and the pricing of financial securities, in: Benjamin Friedman and Frank Hahn, eds., Handbook of monetary economics (North-Holland, Amsterdam).
Newey, Whitney K. and Kenneth D. West, 1987, A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, Econometrica 55, 703–708.
Pagan, A. and Y. Hong, 1991, Nonparametric estimation and the risk premium, in: William A. Barnett, James J. Powell, and George Tauchen, eds., Nonparametric and seminonparametric methods in econometrics and statistics, Proceedings of the fifth international symposium in economic theory and econometrics (Cambridge University Press, Cambridge).
Perlman, Michael D., 1969, One sided problems in multivariate analysis, Annals of Mathematical Statistics 40, 549–567.
Richardson, Matthew P., Paul A. Richardson, and Tom Smith, 1992, The monotonicity of the term premium: Another look, Journal of Financial Economics 31, 97–105.
Roll, Richard, 1977, A critique of the asset pricing theories tests, Journal of Financial Economics 4, 129–176.
Roll, Richard and Stephen A. Ross, 1992, On the cross-sectional relation between expected returns and betas, Working paper (University of California, Los Angeles, CA and Yale University, New Haven, CT).
Schwert, G. William, 1990, Indexes of the United States stock prices from 1802 to 1987, Journal of Business 63, 399–426.
Shanken, Jay, 1987, Multivariate proxies and asset pricing relations: Living with the Roll critique, Journal of Financal Economics 18, 91–110.
Siegel, Jeremy J., 1992, The real rate of interest from 1800–1990: A study of the U.S. and U.K., Journal of Monetary Economics 29, 227–252.
Tauchen, George and Robert Hussey, 1991, Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models, Econometrica 59, 371–396.
Wheatley, Simon M., 1989, A critique of latent variable tests of asset pricing models, Journal of Financial Economics 23, 325–338.
White, Halbert, 1980, A heteroskedasticity consistent covariance matrix estimator and a direct test for heteroskedasticity, Econometrica 48, 817–838.
Whitelaw, Robert, 1992, Time variations and covariations in the expectation and volatility of stock market returns, Working paper (New York University, New York, NY).
Wolak, Frank A., 1987, An exact test for multiple inequality and equality constraints in the linear regression model, Journal of the American Statistical Association 82, 782–793.
Wolak, Frank A., 1989a, Local and global testing of linear and nonlinear inequality constraints in nonlinear econometric models, Econometric Theory 5, 1–35.
Wolak, Frank A., 1989b, Testing inequality constraints in linear econometric models, Journal of Econometrics 41, 205–235.
Wolak, Frank A., 1991, The local nature of hypothesis tests involving inequality constraints in nonlinear models, Econometrica 59, 981–995.
Yancey, Thomas A., Robert Bohrer, and George G. Judge, 1982, Power function comparisons in inequality hypothesis testing, Economics Letters 5, 161–167.