EXPECT THE WORST

Value-at-risk fails to capture all aspects of market risk, argue Jacob Boudoukh, Matthew Richardson and Robert Whitelaw, who present a complementary risk measure – worst-case scenario analysis

Senior management tend to prefer a single measure of the risk of their firm's assets and it is increasingly value-at-risk that is chosen. VAR expresses the price risk of a portfolio of assets in terms of the frequency with which a specific loss will be exceeded. For example, if daily changes in the value of the portfolio are distributed normally, then the firm would expect to lose $\mu_p - 2.33\sigma_p$ on one day out of every 100 (where μ_p and σ_p are the mean and volatility of the portfolio's daily change in value).

VAR is gaining support in the international supervisory community. For example, both the Group of Thirty derivatives committee (November 1994) and the Derivatives Policy Group (March 1995) have called for universal acceptance of this measure. In fact, some products, such as JP Morgan's RiskMetrics, already incorporate VAR directly.

But is VAR really what firms should care about when setting their risk management strategy? We suggest an alternative or complementary measure, related to stress testing: "worst-case scenario" risk. WCS asks the following question: what is the worst that can happen to the value of the firm's trading portfolio over a given period (eg, 20 trading days)?

We build our analysis around a specific assumption. This is that the firm adjusts its portfolio positions over time to maintain the same fraction of capital invested. This is a logical assumption which corresponds to the firm increasing its bet as it makes money and reducing it as it loses.

To understand why WCS may be a more appropriate risk measure than VAR, consider the example above, where the firm's portfolio return is normally distributed with a mean μ_p and volatility σ_p. VAR tells us that losses greater than $\mu_p - 2.33\sigma_p$ will occur, on average, once over the next 100 trading periods, and that losses greater than $\mu_p - 1.65\sigma_p$ will occur, on average, once over the next 20 trading periods. From a risk management perspective, however, managers care more about the size of the losses than the number of times they will face loss.

In contrast to VAR, WCS focuses on the distribution of the loss during the worst trading period (eg, two weeks), over a given horizon (eg, 100 two-week periods). The key point is that a worst period will occur with probability one. The only question is, how bad will it be?

WCS analysis is similar to stress testing and scenario analysis. These concepts are most often associated with the analysis of rare or extreme events, whereas WCS is concerned with the nature of an event which, by definition, is bound to happen.

We will show that the expected loss during the worst period is far greater than the corresponding VAR. Of more importance, there is a substantial probability of a much more severe loss.

Without loss of generality, assume that the change in the value of a firm's portfolio over a trading period is normally distributed, with a mean of 0 and a volatility of 1.[1] Over N of these intervals, VAR states how many times one might expect to exceed a particular loss. In contrast, WCS states what the distribution of the maximum loss will be. That is, it focuses on $F(\min[z_1, z_2, \ldots, z_N])$, denoted $F(\underline{Z})$, where $F(\cdot)$ denotes the distribution function and z_i denotes the normalised return series, corresponding to the change in the portfolio's value over interval i.

Table 1 shows the expected number of trading periods in which VAR will be exceeded. For example, the 5% VAR corresponds to 1.65 in the normalised units in the table and is expected to be exceeded once over a horizon of length 20, and five times over a horizon of length 100.

The table also provides information regarding the WCS measures over different horizons. The distribution is obtained via a simulation of 10,000 random normal vectors (using antithetic variates) of lengths corresponding to the various horizons. For example, the WCS distribution indicates that the expected worst loss over the next 20 periods is 1.86, while over the next 100 periods it is 2.51. More importantly, over the next 20 periods there is a 5% and a 1% probability of losses exceeding 2.77 and 3.26 respectively. The corresponding losses for a 100-period horizon are 3.28 and 3.72 respectively.

Looking at the results from a different perspective, eg, for the 1%, 100-period VAR measure, the VAR is 2.33 while the expected WCS is 2.51 and the first percentile of the WCS distribution is 3.72. If the fraction of capital invested throughout the 100 periods is maintained, then WCS is the appropriate measure in forming risk management policies regarding financial distress. If the firm maintains capital at less than 160% of its VAR, there is a 1% chance that the firm will face financial distress over the next 100 periods.

Next, we discuss two specific examples involving fixed-income securities. Our main purpose is to demonstrate the effect of leverage on VAR and WCS. In table 2 we examine the risk inherent in a position in a 10-year zero-coupon bond and a one-year, at-the-money option on a 10-year zero coupon bond.[2] We assume that the current instantaneous interest rate is 8% a year and the daily volatility of this rate is 7 basis points (these numbers are calibrated from average past data). The bond is thus worth 46 cents per $1 of face amount.

1. Worst-case scenario analysis

The WCS, denoted $\underline{Z}$, is defined as the lowest observation in a vector $Z=(z_1, z_2, \ldots, z_H)$ of length H=5,20,100,250 of independent draws which are normally distributed with mean 0 and volatility 1

	Horizon (H)			
	5	20	100	250
E[Number of $z_i<-2.33$]	0.05	0.20	1.00	2.50
E[Number of $z_i<-1.65$]	0.25	1.00	5.00	12.50
Expected WCS	-1.16	-1.86	-2.51	-2.82
Percentiles of $\underline{Z}$				
1%	-2.80	-3.26	-3.72	-3.92
5%	-2.27	-2.77	-3.28	-3.54
10%	-2.03	-2.53	-3.08	-3.35
50%	-1.13	-1.82	-2.47	-2.78

[1] This assumption corresponds to a normalised portfolio return series, $(R_{p,t+1}-\mu_{p,t})/\sigma_{p,t}$, where the time subscript signifies the fact that both the conditional mean and the volatility may vary over time. See our concluding remarks for a description of how the analysis here may be extended to accommodate deviations from this assumption

2. Worst-case scenario analysis of returns on bonds and bond options

The WCS, denoted $\underline{R}$, is defined as the worst percentage return on (i) a 10-year zero-coupon bond, and (ii) an option with one year to maturity on a 10-year zero-coupon bond, in a vector R of length H=5,20,100,250 of independent returns

Percentage return on bonds	**Horizon (H)**			
	5	20	100	250
Expected WCS	–0.81	–1.29	–1.74	–1.95
Percentiles of $\underline{R}$				
1%	–1.94	–2.25	–2.57	–2.70
5%	–1.58	–1.92	–2.27	–2.45
10%	–1.41	–1.76	–2.13	–2.32
50%	–0.79	–1.26	–1.71	–1.92
Percentage return on bond options	**Horizon (H)**			
	5	20	100	250
Expected WCS	–9.06	–14.28	–18.99	–21.18
Percentiles of $\underline{R}$				
1%	–21.08	–24.19	–27.24	–28.54
5%	–17.37	–20.86	–24.38	–26.11
10%	–15.61	–19.19	–22.97	–24.82
50%	–8.91	–14.05	–18.74	–20.91

Applying VAR, the 1% tail is represented by an increase of 16.3bp [2.33 × 7bp] in interest rates, which results in a loss of 1.62% of the value of the bond, ie, approximately, the 0.163% move in rates multiplied by the duration of the bond, which is 10. The 5% tail is represented by an increase of 11.6bp [1.65 × 7bp] in interest rates, which results in a loss of 1.15% of the value of the bond. The expected WCS loss is 1.74%, while the 1% tail of the WCS distribution is 2.57%.

Now suppose we invest in a one-year option on a 10-year zero-coupon bond. The 1% VAR of holding this option is 17.77% (12.83% for the 5% VAR) and the expected WCS is 18.99%. More importantly, the 1% tail of the WCS distribution corresponds to a loss of 27.24% during a single trading period over the next 100 periods.

Our analysis indicates the importance of the WCS facing a firm, in addition to the firm's VAR. In practice, the WCS analysis has some natural extensions and caveats, which also pertain to VAR.

First, our analysis was developed in the context of a specific model of the firm's investment behaviour, ie, we assumed that the firm, in order to remain "capital efficient", increases the level of investment when gains are realised. There are alternative models of investment behaviour, which suggest other aspects of the distribution of returns should be investigated. For example, we might be interested in the distribution of "bad runs", corresponding to partial sums of length J periods for a given horizon of H.

Second, the effect of time-varying volatility has been ignored. Assuming that risk capital measures are adjusted to reflect this, via RiskMetrics, Garch, density estimation, implied volatility or another method, there is the issue of model risk. That is, to the extent that volatility is not captured perfectly, there may be times when we understate it. Consequently, the probability of exceeding the VAR and the size of the 1% tail of the WCS will be understated.

Third, and related to model risk, there is the issue of the tail behaviour of financial series. It is well established that volatility forecasting schemes tend to understate the likelihood and size of extreme moves. This holds true for currencies, commodities, equities and interest rates (to varying degrees). This aspect will also tend to understate the frequency and size of extreme losses. For our specific case, one could infer a distribution from historical series to obtain a better description of the relevant distribution and so capture the tails. This caveat extends naturally to the issue of correlations, where the most important question, perhaps, is whether extreme moves have the same correlation characteristics as the rest of the data. Of course, if correlations in the extremes are higher, we face the risk of understating the WCS risk.

In conclusion, the analysis of the WCS, and further investigation of the caveats discussed above, is important for the study of some of the more recent proposals on the use of internal models and the more lenient capital requirements imposed on "sophisticated" banks and major dealers. For example, the Basle Committee on Banking Supervision has suggested that banks may elect to use internal models to calculate the VAR inherent in their portfolios. This addresses the call from sophisticated participants for more flexibility in their capital requirements, which will include an allowance for the diversifying effects across various holdings in a global portfolio. They need this because they need to remain "capital efficient". More specifically, sophisticated participants believe that inflexible capital requirements make banks over-capitalised, hence reducing their return on capital.

The new degree of flexibility of the Basle Committee has been met with enthusiasm. It is also widely believed that the degree of flexibility which will be afforded to some financial institutions may result in banks flocking to use internal models, regardless of their true level of sophistication. JP Morgan, for example, introduced a "regulatory dataset" version of RiskMetrics in May 1995, entitled RiskMetrics/RD, which meets the Basle Committee requirements and only varies slightly from the original.

There is one common theme in all these proposals and initiatives. Since VAR incorporates a significant "ruin" probability, be it 5% or 1%, there is a need for an additional layer of prudence via a larger capital requirement. The approach commonly used in the context of the VAR measure is to use the VAR number as an indication, and then simply to multiply this measure by some "hysteria factor", taken out of thin air. A factor of three for example, which is common to many of these approaches, will bring us from the VAR of a portfolio to a capital charge of 3 × VAR.

For example, given a VAR measure of 2.33 standard deviations away from a given portfolio's forecast profit level, the capital charge would be 7 [2.33 × 3] standard deviations away from μ_p. This additional layer of prudence takes into account a number of factors missing in VAR, such as:

- Capitalisation at the 1% VAR will be breached, on average, once every 100 periods, which is too frequent and lenient. Our WCS analysis is especially relevant as a precise measure of prudence.
- There are both estimation risk and model risk in pinpointing volatility and correlation. Our outlined treatment above provides a natural extension to WCS analysis to address this issue.

Our analysis can hence be viewed as a first step on the way to better explaining the "hysteria factor" which should be applied to VAR. ■

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[2] As an illustration, we assume that interest rates follow a simple Vasicek process, ie, dr=σdz and that European options are priced the usual way. See Jamshidian, F, 1989, *An exact bond option pricing formula*, Journal of Finance 44 (1), pages 205–209